Efficient Market Segmentation*

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Abstract

This paper studies the effects of competition on efficiency as well as the different market structures that may arise in equilibrium. Our model is an infinite horizon duopoly with identical firms and homogenous consumers. Firms choose price and effort while consumers care about price and quality – a noisy function of the unobserved effort level exerted by the firm. This imperfect monitoring feature may be disruptive to efficiency. For example, in the presence of a single firm, an equilibrium in which high effort is exerted every period only exists under a special restriction on beliefs off-equilibrium path. We study the tension between this moral hazard problem and the potentially positive effects on reputation generated by competition. First, we show that competition can induce firms to exert high effort every period. In this equilibrium, consumers receive goods of the highest expected quality in every period, which is in contrast to most models of dynamic moral hazard. Second, we show that – under a particular range of parameters – there exists an equilibrium with market segmentation. This equilibrium is characterized by one firm always exerting high effort, thereby producing goods with higher expected quality, whereas the other firm exerts low effort. Moreover, due to the fixed costs of effort, this equilibrium may be Pareto superior to the one in which both firms exert high effort. These two results suggest that competition in a repeated duopoly may be beneficial for efficiency, even if it implies that one firm is very small and specializes in low-quality goods.

JEL classification: C7, D8
Keywords: Reputation, Duopoly, Market Segmentation

1 Introduction

Maintaining a reputation for producing high-quality goods is important in certain markets, such as markets for experience goods. Consumers are willing to pay a premium for a higher quality good, but they can only assess the quality of a good after consuming it. In these markets, a seller benefits from being perceived as a high quality producer. For example, in the market for education, parents may be willing to pay a high tuition if their child gets a high-quality education. In this case, parents can rely on a public history – namely, the quality of previous students. However, they will only be able to assess the quality of the education provided to their own child after graduation

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– or even several years later. Many other markets exhibit the same features, such as restaurants, doctors, personal stylists, mechanics, and many service providers in general.

In such markets, the sellers may want to maintain a reputation for the quality of the good that they provide. A university may target top students; a restaurant owner may choose to be a place for fine dining. These choices often depend on exogenous characteristics of the provider: Not all universities can choose to be top, and not all chefs can cook high-quality food. Such characteristics may be intrinsic to the sellers or may deserve a theory of their own. However, in this paper, we consider the case in which sellers have identical technologies and consumers are homogeneous, with no asymmetric information (thus price discrimination is not an issue). We provide a theory that accounts for the fact that some firms have always had a reputation for high quality, while others have always had one for lower quality, even though they may exhibit the same technology. Most importantly, in our model, when an equilibrium with market segmentation exists, it may be the only efficient equilibrium.

Specifically, we consider an infinite horizon model in which two firms face a continuum of myopic consumers every period. A consumer buys at most a single product per period from one of the firms. The utility of the consumer depends on the price paid and the quality of the good bought. We assume that the consumers must buy the goods before they can experience their quality, which is a noisy signal of the unobservable effort chosen by the firm. Effort can be low or costly and high. In such a market, consumers’ beliefs are valuable: The more they believe that the good will be of high quality, the higher the price that the firm is able to charge. In the benchmark case of a single firm in the market, no equilibrium exists in which it always exerts the costly high effort. The intuition is that if such an equilibrium existed, the consumers would believe that with probability one the firm exerts high effort, thus any bad outcome would be attributed to noise. This, in turn, would generate incentives for the firm to exert low effort, which is a contradiction.

However, if we have competition between firms, then reputation building may be sustained. Loosely speaking, if the fear of losing consumers to competitors after providing a low-quality good exceeds the costs of exerting high effort, firms will have incentives to sustain their reputation. A simple intuition is provided in the seminal work of Klein and Leffler (1981), who focused on equilibria in which once the firm produces low-quality goods, it is out of the market. In their case, if the firm is sufficiently patient, it may choose to never provide low-quality goods. In their model, contrary to ours, quality is not a stochastic function of effort. When it is – and if its inverse relation has full support – their intuition no longer holds; otherwise all firms would eventually be out of the
In this paper, we combine competition with unobservable effort. Thus, we capture the tension between the negative effects on reputation building generated by the noisy signal with the, possibly, positive effect of competition. In particular, we study a dynamic duopoly model with imperfect information, and we show that there is an equilibrium in which goods with high expected quality are always produced. With two competing firms, the consumers’ behavior may discipline the firms, despite the moral hazard problem. The intuition for this is that if consumers punish low quality – which can occur even under high effort – by switching firms, this may generate the necessary incentives for firms to always exert high effort.

Other authors have obtained similar results, showing that competition may generate incentives for firms to always exert high effort. However, those models include incomplete information, and therefore consumers do not get the high quality good (in expectation) every period. We discuss some of these models below.

Our second main result is to show that under a suitably defined range of parameters, an equilibrium with market segmentation exists. Here, one firm specializes in the high-quality goods (in expectation), while the other firm produces the low expected quality good. Consumers know this in equilibrium. They will only buy from the lower quality firm if they pay lower prices. This market segmentation can, in fact, be the only efficient outcome. The intuition is that in this equilibrium only one firm pays the fixed cost of high effort. In contrast, when the two firms produce high expected quality goods, they both pay the fixed cost.

To summarize, we have two main results. First, we show that there exists a pure strategy equilibrium in which both firms always exert high effort. In contrast to most models of dynamic moral hazard, ours does not include incomplete information. This implies that in this high effort equilibrium, consumers get the good with the highest expected quality every period. Second, we show that, under a range of parameters, there is also an equilibrium with market segmentation. Although consumers are homogenous and firms are identical, there is an equilibrium in which one firm produces a good with high expected quality and gets a higher profit, whereas the other firm produces a lower expected quality good. This may be the only efficient equilibrium in the economy.

Our model builds on the work of Mailath and Samuelson (2001), who consider a similar framework; however, contrary to our model, theirs includes a single firm and incomplete information: with positive probability the firm is inherently incapable of exerting high effort (“inept type”). They show that this uncertainty is not enough to generate the necessary incentives for the more
capable firm (“normal type”) to always exert high effort. If such an equilibrium existed, then after a history with an arbitrarily large number of consecutive good outcomes the consumers’ belief that the firm is a normal type would be very close to one. This belief would, then, be very inelastic with respect to the outcomes. As such, the normal type’s incentive would be to exert low effort. Mailath and Samuelson further show that, if in every period there is a probability that the current firm will be replaced by a new firm of unknown type, this is sufficient to generate the incentives to sustain an equilibrium in which the normal type of the firm always exerts high effort. The intuition is that the belief of the consumer is now never too close to one, but is bounded by the replacement probabilities. In the equilibrium just described, the normal type is always exerting high effort, but from the consumers’ perspective, the expected quality of the good is always lower than its maximum due to the incomplete information, which is not the case in our model.

Less related to our paper is the seminal work of Holmstrom (1999), who provides a model in which high effort can be sustained. If effort determines not only the current outcome, but also an observed technology that will in turn help determine the current outcome, then this may be enough to guarantee that high effort is always exerted. In particular, in his model workers exert high effort in order to increase their observed productivity. This productivity is changing over time, which generates a constant incentive for workers to exert high effort. The intuition is similar to Mailath and Samuelson’s (2001) replacement probabilities. This mechanism ensures that the moral hazard problem is solved, but the outcome is not the highest expected outcome, due to the constant uncertainty on the current productivity level.

The effect that competition has on reputation is not straightforward. The main intuition for why a firm may consider exerting high effort in a competitive market is if the expected continuation payoff of a higher reputation is higher than the sum of the current payoff of exerting low effort and the expected continuation payoff of this less costly action. It may be the case that competition drives the firms’ surplus to zero; thus, the discounted value of having a higher reputation becomes small enough such that it does not compensate the short-term gains that a firm may have for deviating to low effort. This was discussed by Bar-Isaac (2005), who argues that competition may have a “non-monotonic” effect on reputational concerns. A single firm, as we previously argued will not exert high effort every period; however, in a duopoly, as in the current paper and in Klein and Leffler’s (1981) work, firms may be induced to exert high effort and maintain a high reputation, whereas in a perfectly competitive environment it may not be possible to sustain these incentives.

Horner (2002) analyzed a dynamic environment with perfect competition. He showed that a
competitive equilibrium exists in which the moral hazard problem is solved. In his model, the moral hazard problem is similar to ours, but he also assumes adverse selection, as in Mailath and Samuelson (2001). In Horner’s model, consumers privately experience the quality of the goods that they purchase, but they observe the “consumer base” of each firm. All consumers can now coordinate their behavior on public histories. Horner focuses on equilibria in which the consumers leave each firm once they experience bad quality; as a result, the firm is driven out of the market once it produces a bad outcome. For every public history, each firm has either always produced good outcomes (having a positive consumer base) or produced a bad outcome at least once (having no consumer base). This leads to an equilibrium in which the normal type always chooses high effort. The intuition of Horner’s result is that the fear of exiting the market disciplines the firm and solves the moral hazard problem. The mechanism in which competition can generate the incentives for producing high quality in our paper is similar to Horner’s (2002) intuition, but under a less severe restriction on the consumer’s behavior after observing a low quality outcome. Moreover, Horner’s model does not lead to vertical differentiation which our model can account for.

This paper is organized as follows. In section 2, we describe the model and the equilibrium concept. We also show that if the economy has a single firm, then high effort every period is not an equilibrium. This will serve as a benchmark to our duopoly study. In section 3, we show that there is an equilibrium in which both firms produce high quality goods every period and another equilibrium with market segmentation under the same parameters as above.

2 Model

Assume that two long-lived firms interact every period in an infinite horizon economy. There is a mass of consumers that we normalize to one. Consumers do not have strategic incentives: they are assumed to be too small to affect the outcomes. Every period each consumer buys at most 1 good from one of the two firms. The consumer’s utility depends on the quality of the good that he bought and on the price that he paid. The quality of the good can be either good or bad, we denote quality by $\omega \in \{g, b\}$. Consumers are risk neutral, identical and we assume a separable utility function. If the quality outcome is $g$, the consumer’s utility at that period is $1 - p$, where $p$ is the price paid by the consumer, while if the outcome is $b$, his utility is $-p$.

Every period the firm is to choose a price, $p \in [0, \infty)$, and an effort level that can be low or high: $e \in \{L, H\}$. To exert high effort the firm must pay a fixed cost $c > 0$, while we normalize the cost of low effort to zero. Besides the fixed cost of high effort, there are no extra costs in producing.
We assume that quality is not persistent: the quality of the good depends only on the current effort level, according to the probabilities described in the table below:

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Table 1

where \(1 \geq α > β > 0\), so that high quality goods are more likely under high effort level. We also assume that \(α − β > 2c\), so that two firms exerting high effort is Pareto superior to the outcome in which both firms exert low effort. The realization of the quality is observed by everyone in the economy at the end of each period. A public history at time \(τ\) is denoted by \(h_τ = \{(ω_{1,τ}, ω_{2,τ}, p_{1,τ}, p_{2,τ})\}_{t=1}^τ\), and the set of all public histories in time period \(τ\) is \(H_τ\). Let \(\{H_τ\}_{t=1}^∞\) be the set of all public histories.

Define a public strategy profile to be a strategy profile in which each player’s actions in each period depend on the public history only. A behavioral public strategy of firm \(i\) at time \(t\) is defined as:

\[
σ_{i,t} : H_τ \to \{H, L\} \times [0, ∞).
\]  

The strategy for each consumer is to choose a probability from which he buys from firm \(i\), firm \(j\) or neither. If he doesn’t buy from any of the two firms, his utility is assumed to be zero. His behavioral strategy is:

\[
s_t : H_τ \times [0, ∞)^2 \to \left\{(s_i, s_j) \in [0, 1]^2 : s_i + s_j \leq 1\right\}.
\]  

Let \(σ\) be the public strategy profile, which is a strategy for each firm, and each consumer. There are multiple Nash equilibria in this game, where at any equilibrium, firms maximize profits, and consumers maximize their utilities. We will concentrate on public equilibria.

**Definition 1** [Perfect Public Equilibrium]

A perfect public equilibrium is a public strategy profile \(σ\) such that for each \(t\) and every history \(h_t\), \(σ\) prescribes a Nash equilibrium in the infinitely repeated game starting at stage \(t\) after history \(h_t\).

The belief that the consumer has at the beginning of each period that the quality of the good produced by firm \(i\) is good, following history \(h_t\), the prices \(p_i\) and \(p_j\) and given the strategy profile \(σ\), will be denoted by \(γ_i (h_t, σ)\).
In any Nash equilibrium, the demand of each consumer \( l \) for the good of firm \( i \) must satisfy the following conditions. First, \( s_{l,i} = 1 \) and \( s_{l,j} = 0 \) if

\[
\gamma_i (h_t, \sigma) - p_i > \gamma_j (h_t, \sigma) - p_j, \quad \text{and}
\]

\[
\gamma_i (h_t, \sigma) - p_i > 0,
\]

while \( s_{l,i} = s_{l,j} = 0 \) if

\[
0 > \max_{k \in \{i,j\}} \gamma_k (h_t, \sigma) - p_k.
\]

Finally, \( s_{l,i} > 0 \) and \( s_{l,j} > 0 \) if

\[
\gamma_i (h_t, \sigma) - p_i = \gamma_j (h_t, \sigma) - p_j \geq 0.
\]

Let's denote \( q_i \) to be the total expected quantity bought from firm \( i \):

\[
q_i = \int_0^1 s_{l,i} (h_t, p_1, p_2) dl.
\]

Consider first the case of a single firm in the economy. There is no equilibria in which the firm exerts high effort every period. The proof is quite simple. If at every period the consumers expect that the firm exerts high effort, then \( \gamma (h_t) = \alpha, \forall h_t \in \mathcal{H}_t \). Then, the firm charges \( p = \alpha \) and all consumers buy from this firm. The expected continuation payoff of following this strategy profile is:

\[
V (\sigma) = \frac{\alpha - c}{1 - \delta}.
\]

If the firm deviates at a particular period, the expected payoff of the deviation is higher in the current period and exactly the same in the next period, since any outcome \( g \) or \( b \) is consistent with the proposed equilibrium strategy.

\[
V (\sigma') = \alpha + \delta \frac{\alpha - c}{1 - \delta} > \frac{\alpha - c}{1 - \delta}.
\]

Therefore, high effort every period cannot be an equilibrium if the market has a single firm. We state this result below.

**Proposition 1 [Single Firm: No high effort equilibrium]**

With a single firm, there is no Nash equilibrium in which the firm produces high quality with probability one every period.

This implies that in any Nash equilibrium of an economy with a single firm, the expected quality of the good is always less than the efficient one: \( \gamma (h) < \alpha \), for any history \( h \in \{\mathcal{H}_t\}_{t=0}^{\infty} \).
3 Market Structure

From now on we will consider the case of a market with two firms. The firms are ex-ante identical in costs and quality technology. However, they have clearly identifiable brands.

First, note that for any discount factor, there is always an equilibrium in which both firms always exert low effort. In particular, for myopic firms, the problem becomes simply a repetition of the static Bertrand model. For sufficiently patient firms, the result is slightly more general and is shown below.

Lemma 1 For any price small enough, \( \forall \tilde{p} \in [0, \beta] \), there exists a perfect public equilibrium strategy profile \( \sigma \) in which both firms always exert low effort and charge price \( p = \tilde{p} \), with the consumer beliefs being \( \gamma(h_t) = \beta \), for any \( h_t \in \{\mathcal{H}_t\}_{t=0}^{\infty} \), and consumer demand satisfying (3a),(3b), and (3c).

Proof. Consider the strategy profile \( \sigma^* \), in which firms exert low effort regardless of the history and charge price \( p = 0 \) every period. Half of the unit mass of consumers buy from firm 1 and half buys from firm 2. \( \sigma^* \) is a perfect public equilibrium, since firms do not have an incentive to deviate: higher price leads to zero profit, as under \( \sigma^* \) and high quality will be interpreted by consumers as an outcome of low effort, so there are no incentives to exert high effort. Each consumer gets a payoff of \( U = \beta \), regardless of the firm that he buys from. The expected payoff for each firm under \( \sigma^* \) is \( V^* = 0 \).

Now consider the following strategy profile: always exert low effort, regardless of the history, and charge \( p = \tilde{p} \) following any history in which both firms have always charged \( \tilde{p} \). If the posted price has ever been different than \( \tilde{p} \), then switch to \( \sigma^* \) thereafter. Then, \( \sigma \) is an equilibrium if

\[
V(\sigma) = \sum_{t=1}^{\infty} \delta^{t-1} q_t \tilde{p} \geq \tilde{p}.
\]

This happens if and only if the equilibrium sequence \( \{q_t\}_{t=1}^{\infty} \) is such that \( \sum_{t=1}^{\infty} \delta^{t-1} q_t \geq 1 \). In particular, consider any stationary sequence in which \( q_t = \bar{q} \in (0, 1) \), such that \( \delta \geq \bar{q} \geq 1 - \delta \).

For sufficiently patient firms and for a sufficiently low cost of exerting high effort, there is also an equilibrium in which both firms exert high effort every period. In this equilibrium, consumers get the good with highest expected quality every period.

Proposition 2 [High Effort Equilibrium]

For any \( \alpha \), and \( \beta \) there exists a discount fact \( \bar{\delta}_{\alpha, \beta} < 1 \) and a cost \( \bar{c}_{\alpha, \beta, \delta} > 0 \) such that for any \( \delta > \bar{\delta} \) and any \( \bar{c} > c \geq 0 \), there exists a perfect public equilibrium in which both firms exert high effort every period and charge a price \( p = \alpha \), on equilibrium path.
Proof. Consider the following strategy for each firm. Exert high effort and charge $p = \alpha$ every period as long as all previous periods have been such that both firms have always charged $p = \alpha$. If at any history at least one of the two firms has charged a price $p \neq \alpha$, then switch to $\sigma^*$ described in lemma 1 above. Every period, consumers’ utility is $U = \alpha - p = 0$. Given that the quality of the product produced by each firm is assumed to be a public signal, consumers can coordinate the behavior on the past outcomes. Consider a strategy profile in which the quantity consumed from each of the two firms is a function of the previous period’s observed quality. There are four possible outcomes every stage game $\xi \in \{gg, gb, bg, bb\}$, where the first term of each pair refers to firm 1’s quality and the second term is firm 2’s quality. Consider a symmetric strategy profile for consumers such that they ‘punish’ the firms that have just produced a worst outcome than their competitors:

$$q_{gb}^i = q_{bg}^i > q_{gg}^i = q_{bb}^i, \quad i = 1, 2,$$

with $q_1^\xi + q_2^\xi = 1, \forall \xi$.

Then, in equilibrium, the expected payoff of the firm depends solely on the previous state of the market. Here we state everything for firm 1, but both firms are identical.

$$V_\xi = q_1^\xi \alpha - c + \delta \left( \alpha^2 V_{gg} + \alpha (1 - \alpha) V_{gb} + \alpha (1 - \alpha) V_{bg} + (1 - \alpha)^2 V_{bb} \right).$$

(6)

Note that the expected continuation payoff for each $\xi$ differs only in the current payoff, which is given by the revenue. Given the restrictions in (5), we have that: $V_{gg} = V_{bb} > V_{gb} = V_{bg}$. Solving the system of four equations (6), we have that:

$$V_\xi = q_1^\xi \alpha + \frac{\delta}{1 - \delta} \left( \alpha^2 q_1^{gg} + \alpha (1 - \alpha) q_1^{gb} + \alpha (1 - \alpha) q_1^{bg} + (1 - \alpha)^2 q_1^{bb} \right) \alpha - \frac{c}{1 - \delta}.$$

(7)

Suppose that a firm decides to deviate and exert low effort. This deviation will imply an immediate gain to the firm that deviates, since it is not paying the fixed cost of high effort. However, the deviation will affect the probability distribution over quality levels, which in turn implies less demand (and thus less revenue) for next period. The expected continuation payoff of such deviation when the market is at state $\xi$ is denoted $V'_\xi$ and given by:

$$V'_\xi = q_1^\xi \alpha + \delta (\beta \alpha V_{gg} + \beta (1 - \alpha) V_{gb} + \alpha (1 - \beta) V_{bg} + (1 - \beta) (1 - \alpha) V_{bb}).$$

(8)

Such deviation will not be worth if $V_\xi \geq V'_\xi, \forall \xi$, which using (7) and (8) is equivalent to:

$$\alpha \left( q_1^{gg} - q_1^{bg} \right) + (1 - \alpha) \left( q_1^{gb} - q_1^{bb} \right) \geq \frac{c}{\delta \alpha (\alpha - \beta)}.$$

(9)
Given that both terms in the l.h.s. of (9) are positive, there must exist a $c$ small enough, such that for any $c < \bar{c}$, there is a perfect public equilibrium that satisfies (9). If (9) holds, firms don’t find it profitable to exert low effort even though this would not change their brand value.

If, however, a firm charges a lower price to capture the market, then both firms will switch to $\sigma^*$ thereafter. Thus, if the firm is sufficiently patient it will not find it profitable to do so. In particular, consider a deviation in which a firm charges a price that is slightly smaller than $\alpha$, and does not exert high effort that period. In the current period all consumers will buy from the deviating firm (lower prices, but same expected quality, given that their beliefs are the equilibrium beliefs). Both firms will revert to $\sigma^*$ and earn a 0 continuation profit: $V_{\xi'} = \alpha$. This deviation will not be profitable if and only if:

$$\alpha \left(1 - q_1^\xi\right) \leq -c + \delta \left(\alpha^2 V_{gg} + \alpha (1 - \alpha) V_{gb} + \alpha (1 - \alpha) V_{bg} + (1 - \alpha)^2 V_{bb}\right).$$

This condition is satisfied if we consider $c$ to be sufficiently small and if we consider only equilibria in which

$$\left(\alpha^2 q_1^{gg} + \alpha (1 - \alpha) q_1^{gb} + \alpha (1 - \alpha) q_1^{bg} + (1 - \alpha)^2 q_1^{bb}\right) \alpha > c.$$ 

One example in which this condition holds is if $q_1^{gg} = q_1^{bb} = \frac{1}{2}; q_1^{gb} = 1, q_1^{bg} = 0$ and $\alpha (\alpha - \beta) > 2c$. This proposed stationary sequence of $\{q_t(\xi)\}_t$, together with the proposed strategy for the firm form a perfect public equilibrium of this game. ■

In the equilibrium described above, firms exert high effort every period and charge $p = \alpha$. All consumers purchase the good every period, $q_1^\xi + q_2^\xi = 1, \forall \xi$, and the expected quality of the goods that they buy is always the highest possible: $\gamma = \alpha$.

At first, it may come as a criticism of this equilibrium that given that both firms charge the same price and exert the same level of effort every period, consumers have no strict incentives to switch firms. Here, they do so since they are indifferent. This implies that a switching cost, or some degree of consumer loyalty would not be consistent with this behavior. However, it is also true, that a small amount of incomplete information would overcome restore our result and give strict incentives for punishing bad outcomes. Similarly, a small fraction of commitment type of consumers, that strictly prefer to consume from the firm that delivered the highest quality level in the previous period, would also be consistent with our results. We are reluctant to make extra assumption on the behavior of the consumers and firms, and we only assume that there are no switching costs or loyal consumers in this market.
The market structure that arises in the equilibrium proposed above is one in which both firms are always exerting high effort. There can be, however, an equilibrium in the same market (i.e. with the same parameters) in which there is an endogenous market segmentation. In this equilibrium, one firm charges a high price and exerts high effort, while the other firm exerts low effort and charges a lower price. For such an equilibrium to exist, both firms must have incentives to follow the specified strategies. For the high effort firm, there must be an incentive for not exerting low effort, which is unobservable. This is the same idea as the previous proposition: the consumers must discipline the firms, through demand shifts after bad draws of outputs. For this to be true, a necessary condition is that the cost is not “too large” as we showed previously. On the other hand, if the punishment for low output is high and the cost is low enough, the firm exerting low effort may want to deviate and exert high effort, so the cost must not be “too low” either. Formally, for every discount factor, there exists a range of costs such that if the costs fall in the range, then an equilibrium with market segmentation exists. Moreover, if the cost falls outside of this range, then it is not possible to sustain such equilibrium with stationary strategies, i.e. strategies that depend only on last period’s market outcome $\xi$.

**Proposition 3** [Market Segmentation]

Fix $\forall \alpha > \beta > 0$, then, there exists a discount factor $\bar{\delta} < 1$ and an interval $[c, \bar{c}]$ such that for $\delta > \bar{\delta}$ and $c \in [c, \bar{c}]$ there exists a perfect public equilibrium in which at every period $t$, one firm exerts high effort while the other firm exerts low effort: $\gamma_1(h_t) = \alpha$ and $\gamma_2(h_t) = \beta$ for every $h_t \in \mathcal{H}_t$.

**Proof.** Consider the strategy profile in which firm 1 charges $\alpha$ every period, firm 2 charges $\beta$ and firms 1 and 2 exert high and low effort every period, in histories in which both firms have charged $\alpha$ and $\beta$, respectively. After a history in which at least one firm has charged a price different than $\alpha$ or $\beta$, then both firms switch to $\sigma^*$, i.e. exert low effort and charge a price of 0.\(^1\) There exists a discount fact $\bar{\delta} < 1$ such that for any $\delta > \bar{\delta}$, neither of the two firms want to charge a different price. We omit the argument, since it is very close to the proof of lemma 1.

Lets restrict attention to equilibria in which the behavior of the consumers follow (5). Lets focus on potential deviations of each firm on the effort level. Note that these deviations will be undetected, since any quality outcome is consistent with the proposed perfect public equilibrium.

Lets restrict attention to stationary equilibria in which the consumer behavior is conditioned on the previous outcome of the market. On equilibrium path, the expected continuation payoff of firm

\(^1\)The argument applies more generally for any pair of prices $p_1$ and $p_2$, such that $\alpha \geq p_1$, $\beta \geq p_2$, and $p_1 = (\alpha - \beta) + p_2$, so that consumers are always indifferent between buying from firms 1 or 2.
1 is:

\[ V_1(\xi) = q_1^c - c + \delta (\alpha \beta V_1(gg) + \alpha (1 - \beta) V_1(gb) + (1 - \alpha) \beta V_1(bg) + (1 - \alpha) (1 - \beta) V_1(bb)) . \]

Define

\[ W = \frac{p_1}{1 - \delta} \left( \alpha \beta q_1^{gg} + \alpha (1 - \beta) q_1^{gb} + (1 - \alpha) \beta q_1^{bg} + (1 - \alpha) (1 - \beta) q_1^{bb} \right) - \frac{c}{1 - \delta}, \]

then we can write:

\[ V_1(\xi) = q_1^c p_1 - c + \delta W. \]

A deviation to low effort implies a payoff of:

\[ V_1'(\xi) = q_1^c p_1 + \delta p_1 \left( \beta \beta q_1^{gg} + \beta (1 - \beta) q_1^{gb} + (1 - \beta) \beta q_1^{bg} + (1 - \beta) (1 - \beta) q_1^{bb} \right) - \delta c + \delta^2 W. \]

Thus, such an equilibrium will exist only if

\[ V_1(\xi) \geq V_1'(\xi), \]

which after some algebra gives us:

\[ \beta \left( q_1^{gg} - q_1^{gb} \right) + (1 - \beta) \left( q_1^{gb} - q_1^{bb} \right) \geq \frac{c}{\delta \alpha (\alpha - \beta)}. \]  

(10)

The same argument from proposition (2) applies here.

It should also be the case that the firm exerting low effort does not have an incentive to deviate.

Its expected continuation profit is given by:

\[ V_2(\xi) = q_2^c \beta + \delta (\alpha \beta V_2(gg) + \alpha (1 - \beta) V_2(gb) + (1 - \alpha) \beta V_2(bg) + (1 - \alpha) (1 - \beta) V_2(bb)) . \]

Solving the system of equations, gives us:

\[ V_2(\xi) = q_2^c \beta + \delta \frac{\beta}{1 - \delta} \left( \alpha \beta q_2^{gg} + \alpha (1 - \beta) q_2^{gb} + (1 - \alpha) \beta q_2^{bg} + (1 - \alpha) (1 - \beta) q_2^{bb} \right), \]

whereas deviating to a high effort implies a payoff of:

\[ V_2'(\xi) = q_2^c \beta - c + \delta (\alpha^2 V_2(gg) + \alpha (1 - \alpha) V_2(gb) + (1 - \alpha) \alpha V_2(bg) + (1 - \alpha) (1 - \alpha) V_2(bb)), \]

or, solving for the system of equations:

\[ V_2'(\xi) = \frac{q_2^c \beta - c + \delta \beta}{1 - \delta} \left( \alpha^2 q_2^{gg} + \alpha (1 - \alpha) q_2^{gb} + (1 - \alpha) \alpha q_2^{bg} + (1 - \alpha) (1 - \alpha) q_2^{bb} \right), \]

\[ + \delta^2 \frac{\beta}{1 - \delta} \left( \alpha \beta q_2^{gg} + \alpha (1 - \beta) q_2^{gb} + (1 - \alpha) \beta q_2^{bg} + (1 - \alpha) (1 - \beta) q_2^{bb} \right) . \]
Thus, a necessary condition for equilibrium is that \( V_2(\xi) \geq V_2'(\xi) \), which will happen if and only if:

\[
\alpha \left( q_2^{gg} - q_2^{gb} \right) + (1 - \alpha) \left( q_2^{bb} - q_2^{bg} \right) \leq \frac{c}{(\alpha - \beta) \delta \beta}.
\]

(11)

If we concentrate on equilibria in which \( q_1^x + q_2^x = 1, \forall x \), then the above condition is equivalent to:

\[
\alpha \left( q_1^{gb} - q_1^{gg} \right) + (1 - \alpha) \left( q_1^{bb} - q_1^{bg} \right) \leq \frac{c}{(\alpha - \beta) \delta \beta}.
\]

(12)

In the example given above, for \( q_1^{gg} = q_1^{bb} = \frac{1}{2} \) and \( q_1^{gb} = 1 - q_1^{bg} = 1 \), we have that:

\[
\frac{1}{2} \leq \frac{c}{(\alpha - \beta) \delta \beta} \Rightarrow c \geq \frac{(\alpha - \beta) \delta \beta}{2}.
\]

Whereas for the second condition:

\[
\frac{1}{2} \geq \frac{c}{(\alpha - \beta) \delta \alpha} \Rightarrow c \leq \frac{(\alpha - \beta) \delta \alpha}{2}
\]

Thus, for such an equilibrium to exist, it is sufficient to have that:

\[
c \in \left[ \frac{(\alpha - \beta) \delta \beta}{2}, \frac{(\alpha - \beta) \delta \alpha}{2} \right].
\]

To look at efficient outcomes, note first that the consumer surplus in the high effort equilibrium is \( U = \alpha - p \), whereas in the market segmentation equilibrium it is: \( U = \alpha - p_H = \beta - p_L \); where the equality comes from the fact that the consumer is free to choose from which firm to buy from. Therefore, the equilibrium with market segmentation will Pareto dominate an equilibrium with both firms exerting high effort, if the total surplus of the firms is higher. In other words, we are looking for conditions under which the firms’ surplus in the market segmentation case is higher than in the high effort equilibrium:

\[
\alpha \beta \left( \alpha q_1^{gg} + \beta q_2^{gg} \right) + \alpha (1 - \beta) \left( \alpha q_1^{gb} + \beta q_2^{gb} \right) + (1 - \alpha) \beta \left( \alpha q_1^{bb} + \beta q_2^{bb} \right) +
\]

\[
+ (1 - \alpha) (1 - \beta) \left( \alpha q_1^{bb} + \beta q_2^{bb} \right) - c
\]

\[
> \alpha (\alpha \beta + (1 - \beta) (1 - \alpha) \beta + (1 - \alpha) (1 - \beta)) - 2c.
\]

This will happen if and only if:

\[
\alpha \beta \left( \alpha q_1^{gg} + \beta (1 - q_1^{gg}) \right) + \alpha (1 - \beta) \left( \alpha q_1^{gb} + \beta \left( 1 - q_1^{gb} \right) \right) +
\]

\[
+ (1 - \alpha) (1 - \beta) \left( \alpha q_1^{bb} + \beta \left( 1 - q_1^{bb} \right) \right) - c
\]

\[
= \beta + (\alpha - \beta) \left( \alpha q_1^{gg} + \alpha (1 - \beta) q_1^{gb} + (1 - \alpha) \beta q_1^{bb} + (1 - \alpha) (1 - \beta) q_1^{bb} \right)
\]

\[
> \alpha - c.
\]

13
Lets define $\gamma$ to be the per period expected demand of firm 1. Formally:

$$\gamma \equiv \alpha \beta q_{1g}^g + \alpha (1 - \beta) q_{1b}^b + (1 - \alpha) \beta q_{1g}^b + (1 - \alpha)(1 - \beta) q_{1b}^b.$$  

Lets restrict attention to stationary perfect public equilibria, i.e. Nash equilibria in which all strategies depend only in the previous period’s public state $\xi$. Then, using (13) we get that an equilibrium with market segmentation will be Pareto superior to the equilibrium in which both firms exert high effort if and only if:

$$\gamma > \frac{\alpha - \beta - c}{\alpha - \beta}.$$  

(14)

The intuition for this result is that, because of the fixed costs of effort, the efficient outcome would be one in which only one firm serves the entire market and exerts high effort every period. We have seen that this cannot be an equilibrium, though. Therefore, the efficient equilibrium is when both firms serve the market, but with high market concentration, i.e. with most consumers buying from the high effort firm. In particular, when the index $\gamma$ is higher than the threshold in (14), the market segmentation outcome Pareto dominates the outcome in which both firms exert high effort.

This can be seen in figure 1 below. The axis represent the per period expected utility of each firm. Note that for intermediate levels of market segmentation, the equilibrium with both firms exerting high effort Pareto dominates the market segmentation outcome.

*Figure 1  Efficient Outcomes*
However, such an equilibrium with high level of market segmentation may or may not exist depending on the parameters. In fact, we show necessary conditions for existence of symmetric stationary perfect public equilibria with high level of market segmentation. The reason why this equilibrium can only exist on a certain range of parameters is that the demand for each firm must exhibit some variance, in order for the strategy profile to be sequentially rational for both firms. In other words, the high effort firm must be punished with a lower demand after a bad quality outcome, but at the same time, if the punishment is too hard, it means that part of the time there is a high demand for the low effort firm, which hurts efficiency.

In particular, we are looking for strategies in which the three conditions below hold at the same time:

\[ \gamma = \alpha \beta q_{1}^{gg} + (1 - \alpha) \beta q_{1}^{bg} + (1 - \alpha) (1 - \beta) q_{1}^{bb} \geq \frac{\alpha - \beta - c}{\alpha - \beta}, \]

which would guarantee that the equilibrium is efficient, and:

\[ \alpha \left( q_{1}^{gb} - q_{1}^{gg} \right) + (1 - \alpha) \left( q_{1}^{gb} - q_{1}^{bb} \right) \leq \frac{c}{(\alpha - \beta) \delta \beta}, \]

with:

\[ \beta \left( q_{1}^{gg} - q_{1}^{bg} \right) + (1 - \beta) \left( q_{1}^{gb} - q_{1}^{bb} \right) \geq \frac{c}{\delta \alpha (\alpha - \beta)}, \]

where these two conditions would guarantee that the strategy is indeed an equilibrium.

Let’s concentrate on symmetric stationary perfect public equilibria, i.e. when \( q_{i}^{gg} = q_{i}^{bb} = \frac{1}{2}, i = 1, 2 \) and \( q_{1}^{gb} = 1 - q_{1}^{bb} = q_{2}^{gb} = 1 - q_{2}^{bb} = x \). Then the above conditions become:

\[ \beta \left( \frac{1}{2} - (1 - x) \right) + (1 - \beta) \left( x - \frac{1}{2} \right) \geq \frac{c}{\delta \alpha (\alpha - \beta)} \]

\[ \alpha \left( x - \frac{1}{2} \right) + (1 - \alpha) \left( \frac{1}{2} - (1 - x) \right) \leq \frac{c}{(\alpha - \beta) \delta \beta} \]

We can write these conditions as:

\[ \frac{1}{2} + \frac{c}{(\alpha - \beta) \delta \beta} \geq x \geq \frac{1}{2} + \frac{c}{\delta \alpha (\alpha - \beta)}, \]  \hspace{1cm} (15)

thus, a necessary condition for this symmetric stationary strategy to be an equilibrium is:

\[ c < \frac{\delta \alpha (\alpha - \beta)}{2}. \]  \hspace{1cm} (16)

Using symmetric stationary strategies, we have that the level of market segmentation \( \gamma \) is given by:
\[
\gamma = \frac{\alpha \beta + (1 - \alpha)(1 - \beta)}{2} + \alpha (1 - \beta) x + (1 - \alpha) \beta (1 - x).
\]

Therefore we can write:

\[
\begin{align*}
\gamma &= \frac{\alpha \beta + (1 - \alpha)(1 - \beta)}{2} + \alpha (1 - \beta) x + (1 - \alpha) \beta (1 - x) \\
&\geq \frac{\alpha \beta + (1 - \alpha)(1 - \beta)}{2} + \alpha (1 - \beta) \left( \frac{1}{2} + \frac{c}{(\alpha - \beta) \delta \alpha} \right) + (1 - \alpha) \beta \left( 1 - \frac{1}{2} - \frac{c}{\delta \alpha (\alpha - \beta)} \right) \\
&= \frac{1}{2} + \frac{c}{\alpha (\alpha - \beta) \delta} (\alpha (1 - \beta) - \beta (1 - \alpha)) \\
&= \frac{1}{2} + \frac{c}{\alpha \delta}.
\end{align*}
\]

This level of market segmentation will be Pareto superior to the high effort equilibrium if:

\[
\frac{1}{2} + \frac{c}{\alpha \delta} \geq \frac{\alpha - \beta - c}{\alpha - \beta},
\]

which will be true if and only if:

\[
\begin{align*}
(\alpha - \beta) \alpha \delta + 2c(\alpha - \beta) &\geq 2\alpha \delta (\alpha - \beta - c), \\
2c(\alpha - \beta + \alpha \delta) &\geq \alpha \delta (\alpha - \beta).
\end{align*}
\]

Thus, a sufficient condition for a stationary perfect equilibrium that Pareto dominates a high effort equilibrium is:

\[
c \geq \frac{\alpha \delta (\alpha - \beta)}{2 (\alpha - \beta + \alpha \delta)}.
\]  \hspace{1cm} (17)

Therefore, a symmetric strategy profile described above will be an equilibrium if and only if it satisfies (15) and the parameters are such that (16) and (17) hold at the same time.

4 Conclusion

In this paper, we have studied the effects of competition on the firms’ reputation incentives for producing goods with high expected quality. In particular, we focused on the tension between the negative effects of imperfect monitoring on reputation and the positive effects of competition on reputation incentives. We have shown that if the cost of effort is not prohibitively large, there will be an equilibrium in which both firms exert high effort every period. This equilibrium Pareto dominates the equilibrium in which firms never exert high effort. Thus, competition can generate
the necessary incentives for reputation. Although this result is not entirely new, we have shown a different channel through which it can occur. Buyers are always indifferent, a result from the Bertrand competition between the two firms, and can punish low quality by switching firms. Most importantly, we have shown that for a particular range of parameters there is also an equilibrium in which one firm specializes in high expected quality goods and the other firm specializes in low expected quality goods. This equilibrium Pareto dominates the equilibrium in which both firms exert high effort. The intuition here is that high effort requires a fixed cost, which in the equilibrium with market segmentation only one firm pays.

Our result supports the view that competition is in general beneficial for reputation, even if it implies that one firm is very small and specializes in low-quality goods.

References


