Estimating the elasticity of intertemporal substitution: Is the aggregate financial return free from the weak instrument problem?

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Abstract

The elasticity of intertemporal substitution (EIS) is one of the key parameters in the Economics and Finance literature. It is usually estimated by means of the consumer’s Euler Equation using an instrumental variable approach, and the estimates are usually zero or close to zero. Nevertheless, such attempts present two major problems: first, the use of weak instruments, and second, the absence of a rate of return that is representative of the agent’s asset portfolio. The latter has been addressed by using the return of a synthetic mutual fund (SMF), which is a weighted combination of the returns of all assets held by the average household. The use of SMF returns led to EIS estimates of about 0.2 for the US economy. In this paper, we first investigate whether the EIS estimates using the SMF returns for the US suffer from the weak instrument problem. Next, we conduct robustness analyses using different estimators and instrument sets. Our findings show that estimates using SMF returns are plagued by weak instruments, but in some cases partially robust estimators were able to deliver a positive and statistically significant EIS estimate. Furthermore, we found that the Treasury Bill return does not suffer from weak instruments, but the EIS is not precisely estimated and seems to be close to zero.

1. Introduction

For several years, researchers have struggled to obtain consistent estimates of the elasticity of intertemporal substitution (EIS). This parameter is crucial not only in Economics but also in Finance. For instance, in the consumption and portfolio choice problem discussed in Campbell and Viceira (1999), the EIS is the key parameter in the optimal consumption rule.

The theoretical framework often used to estimate the EIS comes from the idea that consumers smooth consumption in order to maximize their lifetime utility. Nevertheless, as pointed out by Browning and Crossley (2001), the researcher needs to choose a particular setup in order to arrive at an empirical specification that can be estimated. The specification often used throughout the literature is depicted by Eq. (1), hereafter called ‘basic setup’,

\[ \ln(C_t) = \alpha_i + \psi r_{it} + \epsilon_{it}, \quad i = 1, 2, \ldots, N \]  

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where \( C_t \) is the per capita consumption in period \( t \), \( x_t \) is the constant term, \( r_{it} \) is the log gross return of the \( i \)th asset held by the consumer in period \( t \), and \( \varepsilon_{it} \) is the error term in period \( t \).

Thus, consumption growth depends on the asset return. The asset return coefficient \( (\psi) \) is the EIS, which measures the agent’s willingness to substitute consumption over time. Usually, the error term \( \varepsilon_{it} \) is assumed to be correlated with the regressor, \( r_{it} \), precluding the use of the ordinary least squares (OLSs) estimator; however, if a matrix of valid instruments \( Z_t \) is available, then an instrumental variable (IV) approach can estimate the EIS.

Several studies estimated Eq. (1) using US data, including Mankiw (1981), Hall (1988) and Campbell and Mankiw (1989). They found estimates of EIS that were small (i.e., below 0.25) and barely statistically significant. As a result, researchers began to examine this issue from different angles.

One angle is to investigate whether the poor performance of the basic setup is due to problems with the econometric methodology, such as the use of weak instruments. Along these lines, Yogo (2004) used the basic setup and showed that the weak instruments problem could be a reason behind these results. In fact, his findings for several OECD countries indicate that weak instruments plagued most of his estimates. By applying econometric techniques that are robust to this problem, Yogo (2004) concluded that the EIS is not significantly different from zero. Gomes and Paz (2011) further scrutinized Yogo’s (2004) results using different instrument sets, and whenever instruments were not weak, Yogo’s results were corroborated.

Dacy and Hasanov (2011), hereafter ‘DH’, studied this problem from a different perspective. They argued that any single asset return may be inappropriate to measure the portfolio return of the representative agent, since agents generally have several assets in their portfolio, such as real estate, bonds and stocks. Thus, they built an aggregate asset named the ‘synthetic mutual fund’ (SMF). The SMF return is the weighted average of the assets returns in the household portfolio, using the assets share as the weight. They also accounted for taxes on the asset returns, in order to use the after-tax return, which has been overlooked by almost all of the literature so far. Their key finding is that when using the SMF return net of taxes, the estimates of the EIS are about 0.20 and statistically significant.

In this paper, we combine Yogo’s (2004) insight regarding weak instruments and DH’s (2011) idea of using the SMF return to estimate the EIS for the US economy. Our first contribution is a careful investigation of the results obtained by DH (2011) and Yogo (2004) for two cases.\(^2\) In the first case, we assume that the error term is homoscedastic and we use Yogo’s (2004) methodology to investigate whether the estimates based on SMF are plagued by weak instruments. For the heteroscedastic error case, we follow Kleibergen (2005) and employ the weak identification robust generalized method of moments continuously updated estimator (CUE-GMM). We conduct hypothesis tests that are robust to weak identification, such as the S, KLM, and JKL for the estimated EIS. Finally, we also compute weak identification robust confidence intervals for the EIS using the Stock and Wright’s (2000) S and Kleibergen’s (2005) KLM statistics.

The second contribution of our study is to check the robustness of DH’s (2011) and Yogo’s (2004) results by using different estimators, instrument sets, and asset returns. So, this exercise also allows us to obtain estimates using Yogo’s (2004) specifications for a different sample, and to evaluate whether the use of after-tax returns leads to better EIS estimates.

Our main findings are as follows: (i) the estimates using the SMF returns suffer from the weak instrument problem for both instrument sets used; (ii) some of these EIS estimates using partially robust estimators were significantly different from zero, positive, and at the low end of DH’s (2011) estimates; (iii) although the specifications using the T-Bill return are not plagued by weak instruments, the EIS estimates are in most cases not statistically different from zero. Finally, the weak instrument robust confidence intervals either included zero or were \((-\infty, +\infty)\), being uninformative and, as a consequence, the weak instrument problems prevent us from drawing a definite conclusion regarding the true value of the EIS. Last, it is worth mentioning that, although Yogo’s (2004) instruments had a slightly better performance than DH’s (2011) instruments in the weak instrument tests, Yogo’s (2004) results do not change when DH’s (2011) dataset is used.

The paper is organized as follows. In the next section, the microfoundations for the estimated specification are provided along with a brief literature review. Section 3 presents the econometric methodology and tests. Section 4 describes the data. Results are discussed in Section 5. Finally, Section 6 presents our conclusions.

2. Theoretical model and literature review

In this section, we discuss a theoretical model that provides the microfoundations and highlights the auxiliary assumptions needed for the derivation of Eq. (1). Next, we present a brief literature review focusing on other studies’ results that are related to this paper.

2.1. Theoretical model

In the lifecycle theoretical framework, consumers maximize their expected lifetime utility by smoothing consumption and the EIS is a key parameter in this optimization problem. In order to arrive at an empirical specification that can be used to estimate the EIS, further assumptions are needed (Browning and Crossley, 2001). The most common assumptions are a

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1. This specification will be discussed in great detail in the next section.
2. In this paper, we are only interested in Yogo’s (2004) results for the US economy.
frictionless economy with a single representative agent, whose preferences are represented by the constant relative risk aversion (CRRA) utility, as done in DH (2011),

\[
\max_{\{C_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \delta U(C_t) \right], \quad 0 < \delta < 1
\]

\[
U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad \text{if } \gamma \neq 1, \quad \text{or} \quad U(C_t) = \ln(C_t) \quad \text{if } \gamma = 1
\]

\[
\text{s.t. } A_{t+1} \leq R_{t+1}(A_t + Y_t - C_t)
\]

where \( \mathbb{E}_0[\cdot] \) denotes the conditional expectation given the information available at time 0, \( \delta \) is the agent’s intertemporal discount rate, \( C_t \) is the consumption level at time \( t \), and \( A_t \) is the agent’s wealth invested in an asset that provides a gross return of \( R_{t+1} \). The coefficient of relative risk aversion is \( \gamma \); and in a CRRA utility function, the EIS is its reciprocal, i.e., \( \psi = \gamma^{-1} \). The solution of the consumer problem leads to the following Euler Equation:

\[
E_{t-1} \left[ \delta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t \right] = 1, \quad i = 1, 2, \ldots, N.
\]

(3)

In the macroeconomics and finance literature (e.g., Cochrane, 2005), the term \( M_t \equiv \delta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \) is called the ‘stochastic discount factor’ (SDF) and Eq. (3) is called the ‘Asset-Pricing Equation’, which should also hold for any asset. Under the assumption that \( M_t \) and \( R_t \) are jointly distributed as a log-normal conditional on \( t-1 \) information, Eq. (3) becomes

\[
\exp \left\{ E_{t-1}[m_t + r_t] + \frac{1}{2} \sigma_{t-1}^2 \right\} = 1
\]

(4)

where \( m_t \equiv \ln(M_t) \), \( r_t \equiv \ln(R_t) \), and \( \sigma_{t-1}^2 = \text{Var}_t - 1[m_t + r_t] \) is the conditional variance. This variance term represents the precautionary savings, and it is subscripted by \( t-1 \) because we do not restrict \( \sigma_{t-1}^2 \) to be constant. Under the rational expectations hypothesis, the expected value at time \( t-1 \) of the error term at time \( t \) is zero, i.e., \( E_{t-1}[\tilde{\epsilon}_{t|t}] = 0 \), and by taking logs of both sides of (4), we obtain

\[
m_t = -r_t - \frac{1}{2} \sigma_{t-1}^2 + \tilde{\epsilon}_{t|t}.
\]

(5)

Rewriting \( m_t \) in terms of consumption variables, we obtain the log-linear time series representation for the consumption growth rate, with \( \tilde{\epsilon}_{t|t} = -\psi \tilde{\epsilon}_{t|t} \).

\[
\ln(c_t) = \psi \ln(\delta) + \psi r_t + \frac{1}{2} \psi \sigma_{t-1}^2 + \tilde{\epsilon}_{t|t}, \quad i = 1, 2, \ldots, N.
\]

(6)

Notice that Eq. (6) is compatible with Eq. (1) regardless of the assumption made about \( \sigma_{t-1}^2 \), because if it is homoscedastic, it will be included in the constant term. Otherwise, it will be part of the error term and then an estimation method that can cope with heteroscedastic errors is required.

On the other hand, Yogo (2004) justifies the use of Eq. (1) by assuming that consumers have the Epstein and Zin (1989, 1991) non-expected utility that is defined by

\[
U_t = \left[ (1 - \beta) c_t^{1-\gamma} + \beta (E_t(U_{t+1}^{1-\gamma}))^{1-\gamma} \right]^rac{1}{1-\gamma}
\]

(7)

where \( \theta = (1 - \gamma)/(1 - \psi^{-1}) \) and the coefficient of relative risk aversion, \( \gamma \), is no longer the reciprocal of the EIS of consumption, \( \psi \). Under a budget constraint similar to the one used in Eq. (2), we can derive the Euler Equation depicted by Eq. (8),

\[
E_{t-1} \left[ \beta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} B_t^{\psi-1} R_t \right] = 1
\]

(8)

where \( B_t \) is the market portfolio return. Following the same steps used in the previous case, we arrive at:

\[
\ln(c_t) = \psi \ln(\beta) + \frac{\psi(\theta - 1)}{\theta} b_t + \psi r_t + \frac{\psi}{2\theta} \sigma_{t-1}^2 + \tilde{\epsilon}_{t|t}, \quad i = 1, 2, \ldots, N
\]

(9)

where \( b_t \equiv \ln(B_t) \) and \( \tilde{\epsilon}_{t|t} = -\frac{\psi}{\theta} \tilde{\epsilon}_{t|t} \). When \( \theta = 1 \), Eq. (1) becomes a special case of Eq. (9). As a result, the coefficient of relative risk aversion becomes the reciprocal of the EIS, i.e., \( \psi = 1/\gamma \). If \( r_t \) is proxied by the market portfolio return, \( b_t \), then the sum of the second and third terms in the right-hand side of Eq. (9) becomes \( \psi b_t \). Hence, Eqs. (1) and (9) become very similar even when \( \theta \) is different from one. As discussed by Epstein and Zin (1991), from the point of view of the non-expected utility model, the predictions of the expected utility model should be hard to reject using the market portfolio to represent the agent’s portfolio, but they could be rejected when other assets are used; however, using the representative agent setup, the portfolio

\[\text{Without the log-normality assumption, Eq. (1) holds as a second-order log linear approximation of the non-linear Euler Equation.}\]
return of the agent and the return of the market can be viewed as the same. Remember that the SMF represents the average household portfolio, so we should not expect a rejection of the specification using the SMF return.

2.2. Literature review

Although not explicitly derived, the rate of return and the consumption levels are simultaneously determined, so the use of OLS does not provide consistent estimates. Thus, the EIS can only be identified if we use a matrix of valid instruments \((Z_t)\). These instruments cannot be correlated with the error term and have to be correlated with the rate of return; in other words, the latter implies that the instruments need to be able to predict the endogenous regressor. The lags of the consumption growth, inflation, and nominal rate of return, among others, have been traditionally used as instruments (see, e.g., Yogo, 2004; DH, 2011). Indeed, due to aggregation problems in quarterly data, the use of lags of variables no closer than the second lag has been recommended by Hall (1988).

Many authors have estimated Eq. (1) using two-stage least squares (TSLs) for the US economy. Mankiw (1981) estimated the EIS around 0.25, being different from zero at the 10% significance level. Hall (1988) estimated Eq. (1) reaching the conclusion that the EIS may be lower than 0.2 and probably zero.

Under the assumption of CRRA preferences, some researchers estimated the reverse of Eq. (1), as shown by Eq. (10). In this case, when the EIS is close to zero, the risk aversion parameter, i.e., the consumption growth coefficient \((\gamma)\), should be very high.

\[
r_{it} = \mu + \gamma \ln(c_{it}) + \eta_{it}, \quad i = 1, 2, \ldots, N (10)
\]

For the US economy, Hansen and Singleton (1983) used a maximum likelihood estimator to estimate Eq. (10) and obtained imprecise point estimates of the coefficient of relative risk aversion that were between zero and two. Campbell and Mankiw (1989) estimated both Eqs. (1) and (10) by means of TSLs. In the first case, they found fairly small values for the EIS. By estimating Eq. (10), they did not find extremely large estimates of risk aversion coefficient as expected; instead, the estimates were clustered around one. Campbell (2003) rejected that the EIS is equal to one by estimating Eq. (1), whereas by means of Eq. (10) the null hypotheses that the risk aversion is one could not be rejected.5

One possible explanation for these puzzling results could be the CRRA preference assumption is wrong. For instance, in the case of Epstein–Zin preferences, Eq. (10) will not imply that the consumption growth coefficient is just the reciprocal of the EIS; however, even when the CRRA utility is not used, the coefficient of Eq. (1) should be close to the reciprocal of the coefficient of Eq. (10).6

While part of the literature move toward other setups, some authors conducted a deeper analysis, asking if the poor performance of the basic setup is due to the failure of other auxiliary assumptions – namely, weak instruments and the choice of the asset return that mimics the return faced by the typical consumer. In our paper, we try to connect these two branches of the literature.

The weak instrument problem occurs when the instruments are mildly correlated with the endogenous variable, and this leads to biased estimates of the EIS.7 Neely et al. (2001) and Campbell (2003) note that both consumption growth and asset returns are difficult to predict and thus the weak instrument problem is likely in estimating the EIS, which could explain the incompatible results between Eqs. (1) and (10) estimates.

Using data from eleven developed countries, Yogo (2004) estimated the EIS taking into account the weak instrument problem. His results imply that, first, the excluded instruments do a poor job predicting the stock return, so estimates that use the stock return are biased and their standard errors are underestimated. Second, although the instruments can predict the T-Bill return rate, the estimated EIS is close to zero. As a result, Yogo (2004) concluded that the EIS is less than 1 and not significantly different from 0 for eleven developed countries.

Gomes and Paz (2011) extended Yogo’s (2004) analysis using other sets of instruments to estimate Eqs. (1) and (10). They confirmed that only the T-Bill return rate is predictable; and these estimates presented rejections of the null of the Sargan overidentification test for the specifications using US data. These rejections cast doubts on either instrument validity or model specification. Hence, the importance of investigating weak instruments is undisputable.

DH (2011) took a different path by examining if the problematic EIS estimates were caused by the use of wrong measures of the asset return faced by the typical consumer. Their point comes from the fact that the theoretical model implies the use of the return of a representative portfolio and not the return of a risk-free asset or stocks like the US Treasury Bill or the S&P 500 index. Thus, they built a market return (SMF) that approximates the observed portfolio choice made by the average household based on money, Treasury Bills, intermediate and long-term government bonds, municipal tax-exempt bonds,
corporate AAA-rated bonds, common stocks, and residential real estate returns. The weights are the market-based shares of each asset in the household portfolio. Indeed, by estimating Eq. (1) with the SMF, DH’s (2011) estimated EIS are statistically significant and range from 0.10 to 0.32.

3. Econometric methodology

The econometric methodology used to estimate the EIS is presented in this section. First, we discuss the necessary conditions to identify the EIS and the available econometric tests to assess if such conditions are met. Next, we introduce the use of some estimators that are partially robust to weak instruments. Finally, we present methods that provide weak instruments robust confidence intervals for the EIS.

As discussed before, the EIS is estimated using the regression model described by Eq. (1), in which the asset return is an endogenous regressor. Thus, we need to use an instrumental variable (IV) method, and this requires the use of excluded instruments. The consistency of the estimator employed depends upon the excluded instruments meeting two necessary conditions. The orthogonality condition requires instruments to not be correlated with the error term. The second condition is that the instruments must be relevant, i.e., they have to be correlated with the rate of return, in particular this correlation cannot be small, otherwise the instruments will be weak, the model will be weakly identified, and the estimates will not be consistent.

To prevent such problem, researchers have used two approaches: test if both necessary conditions are fulfilled and employ weak instrument robust methods. The econometric tests used in the first approach are pre-estimation weak instrument tests and the post-estimation Sargan test for the orthogonality condition. A drawback of pretesting is that with conventional inference methods the size of such two-step testing procedures cannot be controlled (see Kleibergen and Mavroeidis, 2008). The second approach does not have the previous problem but the weaker the instruments the less informative are the weak instrument robust estimates and confidence sets for the EIS, as discussed in Yogo (2004). In sum, the prudent researcher should consider and use both approaches as complements.

3.1. Pre and post estimation tests

The pre-estimation tests available can only detect if instruments are weak, i.e. they are not able to predict the rate of return used. In the presence of weak instruments, the TSLS bias can be large, even in very large samples. As Murray (2006) points out, the confidence intervals computed for TSLS estimates can be very misleading because their mid-point is biased and their width is too narrow, which undermines hypothesis tests based on TSLS.

The first pre-estimation test comes from Kleibergen and Paap (2006), hereafter called ’KP’, and its null hypothesis is that the model is underidentified, i.e. the instruments have no correlation with the endogenous regressor. Yet, underidentification is an extreme case, a rejection of the null of the KP test does not rule out the case of weak instruments that are not extremely uncorrelated with the endogenous regressor.

Under the assumption that the error terms are homoscedastic and serially uncorrelated, Yogo (2004) calculated the first-stage F-statistic of the TSLS, i.e. the endogenous regressor is regressed on the included and excluded instruments.9 And this F-statistic is used to conduct the four weak instrument tests proposed by Stock and Yogo (2002) that have the following null hypotheses: (i) the bias of the TSLS, as a fraction of OLS bias, is greater than 10%; (ii) the bias of Fuller-k, as a fraction of OLS bias, is greater than 10%; (iii) the actual size of the TSLS t-test, at 5% significance, can be greater than 10%; and (iv) the actual size of the LIML t-test, at 5% significance, can be greater than 10%.10

For each null hypothesis, the calculated F-statistic is contrasted with the specific critical value provided by Stock and Yogo (2002). This critical value indicates the lower bound for the first-stage F-statistic, in order to reject each specific null hypothesis. By doing so, the researcher finds whether the estimates are biased or their t-tests present size distortion. For instance, the results of the TSLS estimation of Eq. (1) are unreliable due to weak instrument if the null hypotheses (i) and (iii) are not rejected.

To assess the orthogonality condition we use the Sargan overidentification test. This post-estimation test’s null hypothesis is that instruments are uncorrelated with the error term. However, the null hypothesis can be rejected for two conceptually different reasons: the model being tested is correctly specified but instruments are correlated with error term (i.e. invalid) or the model is not correctly specified. The second possibility occurs, for instance, if the linearized version of the Euler Equation does not depict the true relationship between consumption growth and rate of return. Furthermore,

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8 Mulligan (2002) points out that previous studies tried to estimate the EIS and yielded poor results because the researchers relied on improper asset returns, such as bonds and stocks. He argues that the appropriate return for the representative agent would be given by an after-tax return on aggregate capital. Although conceptually similar, Mulligan’s idea is different from DH (2011) suggestion for using the average household portfolio after-tax returns.

9 Under homoscedasticity and absence of serial correlation, the first-stage $F$ statistic used for testing if the included and excluded instrument coefficients of the first-stage regression are jointly equal to zero is closely related to the concentration parameter ($|\mu^2|$), which measures the relevance of instruments (Yogo, 2004). When $\mu^2$ is large, the TSLS estimator is approximately unbiased, and the distribution of its $t$-statistic is approximately standard normal. However, when $|\mu^2|$ is small, the TSLS can be severely biased and the distribution of its $t$-statistic can be highly skewed (Stock et al., 2002).

10 These weak instrument test statistics have non-standard distributions. Stock and Yogo (2002) tabulated their p-values for different bias and test sizes. We chose the lowest bias size (10%) and the lowest test size (10%) available.
Theorem 3 of Staiger and Stock (1997) argues that under weak instruments the Sargan test can be invalid because the Sargan statistic is based upon the estimated parameter, which is inconsistently estimated in the presence of weak instruments.

3.2. Partially robust estimators

Besides the TSLS, Yogo (2004) considered the Fuller-\( k \) and the LIML estimators because they and their corresponding Wald statistics have non-standard limiting distributions that differ from one another when instruments are weak. Stock and Yogo (2002) make the point that, although both the TSLS and Fuller-\( k \) are biased when instruments are weak, the Fuller-\( k \) bias is less severe for a given population parameter, and the LIML size distortion is less severe than that of the TSLS Wald test. Therefore, the Fuller-\( k \) and LIML can be viewed as estimators that are more robust to weak instruments than TSLS (Stock et al., 2002).\(^\text{11}\)

An empirical strategy based on the previously discussed diagnostic tests and estimators can be very informative regarding the quality of the instruments. Nevertheless, when the error term is heteroscedastic, the efficient estimator is now the GMM as and discussed in Olea and Plueger (2011) the Stock–Yogo critical values for the four pre-tests may not be appropriate. These considerations suggest that the heteroscedastic case is important.

In the GMM framework, the \( 2 \times 1 \) parameter vector \( \beta = \begin{bmatrix} x_i & \psi \end{bmatrix} \) of Eq. (1) is identified by the conditional moment condition \( E[\ln(C_i) - x_{i0} - \psi \theta Z_i] = 0 \), where \( x_{i0} \) and \( \psi \) are the true value of the parameters. A necessary condition for identification is instrument relevance, which implies \( E[\ln(C_i) - x_{i0} - \psi \theta Z_i] = 0 \) for \( \beta \neq \beta_0 \). However, if instruments are weak, the moment condition will be nearly zero for \( \beta = \beta_0 \). This implies that the model is weakly identified, so the criterion function is insensitive to modest changes in the parameters, i.e. it is nearly flat.

Hansen et al. (1996) found evidence that GMM-type estimators behave differently in the presence of weak identification, in particular the continuous updating estimator (CUE-GMM) is less biased, and its confidence intervals have better coverage rates than the two-step GMM. Stock et al. (2002) provides further evidence that weak identification can lead to different confidence sets when the same model is estimated by two-step GMM and iterated GMM, for example. Hence, they suggest that one symptom of weak identification is GMM estimates that are sensitive to additions to the instrument set or to changes in the sample. As a result, the weak identification literature has focused on the CUE-GMM estimator, and this is the estimator used here too.

3.3. Fully robust methods

As discussed before, weak instruments lead to biased point estimates of the EIS, which in turn compromise all tests based on such estimates. To avoid this problem, the last econometric approach used in this paper consists of obtaining confidence intervals for the EIS that are robust to weak instruments. Such intervals are obtained by inverting econometric tests with Wald statistics have non-standard limiting distributions that differ from one another when instruments are weak. Stock and Yogo (2002) make the point that, although both the TSLS and Fuller-\( k \) are biased when instruments are weak, the Fuller-\( k \) bias is less severe for a given population parameter, and the LIML size distortion is less severe than that of the TSLS Wald test. Therefore, the Fuller-\( k \) and LIML can be viewed as estimators that are more robust to weak instruments than TSLS (Stock et al., 2002).\(^\text{11}\)

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An empirical strategy based on the previously discussed diagnostic tests and estimators can be very informative regarding the quality of the instruments. Nevertheless, when the error term is heteroscedastic, the efficient estimator is now the GMM as and discussed in Olea and Plueger (2011) the Stock–Yogo critical values for the four pre-tests may not be appropriate. These considerations suggest that the heteroscedastic case is important.

In the GMM framework, the \( 2 \times 1 \) parameter vector \( \beta = \begin{bmatrix} x_i & \psi \end{bmatrix} \) of Eq. (1) is identified by the conditional moment condition \( E[\ln(C_i) - x_{i0} - \psi \theta Z_i] = 0 \), where \( x_{i0} \) and \( \psi \) are the true value of the parameters. A necessary condition for identification is instrument relevance, which implies \( E[\ln(C_i) - x_{i0} - \psi \theta Z_i] = 0 \) for \( \beta \neq \beta_0 \). However, if instruments are weak, the moment condition will be nearly zero for \( \beta = \beta_0 \). This implies that the model is weakly identified, so the criterion function is insensitive to modest changes in the parameters, i.e. it is nearly flat.

Hansen et al. (1996) found evidence that GMM-type estimators behave differently in the presence of weak identification, in particular the continuous updating estimator (CUE-GMM) is less biased, and its confidence intervals have better coverage rates than the two-step GMM. Stock et al. (2002) provides further evidence that weak identification can lead to different confidence sets when the same model is estimated by two-step GMM and iterated GMM, for example. Hence, they suggest that one symptom of weak identification is GMM estimates that are sensitive to additions to the instrument set or to changes in the sample. As a result, the weak identification literature has focused on the CUE-GMM estimator, and this is the estimator used here too.

\(^{11}\) Considering the tests for weak instruments (i) and (ii) from Section 3, the critical value for TSLS is greater than the critical value for Fuller-\( k \), which is a reflection of the fact that the latter is more robust to weak instruments. Analogously, LIML has a lower critical value than TSLS, being less prone to size distortion (Yogo, 2004).
4. Dataset

The US dataset used in this paper is identical to Dacy and Hasanov’s (2011) data. It consists of quarterly observations from the first quarter of 1952 till the last quarter of 2000. There are two series of real per capita consumption: consumption of nondurables (NDs) and consumption of nondurables plus services (NDSs). The Fisher real asset returns considered are the Treasury Bill (T-Bill), S&P 500 (stock), and the synthetic mutual fund (SMF). For each asset, we use a before-tax and a net-of-tax series.12

The SMF return is constructed as a weighted average of the following asset returns: money (M2), T-Bills, treasury notes, treasury bonds, municipal bonds, corporate bonds, stocks, and housing. The weights come from the household holding information published in the Flow of Funds Accounts (FFAs) edited by the US Board of Governors of the Federal Reserve System (2003).

Additionally, we use the following variables from DH’s (2011) dataset: the bond default yield premium, which is defined as the ratio between long-term corporate bond yields and long-term government bond yields, and the bond horizon yield premium, which is given by the ratio between long- and short-term government bond yields.

Table 1 displays the descriptive statistics of the asset returns and their correlation with the measures of consumption growth.13 The SMF average return is between the T-Bill and the stock average returns. When taxes are taken into account, the average returns become smaller. The correlation between returns and consumption growth is in the 0.1–0.2 range, and the correlation with the ND is larger than for NDS. Interestingly, the after-tax returns have a higher correlation with consumption growth. This increase is particularly substantial for the T-Bill.

5. Results

In this section, we first investigate the homoscedastic case. Following the methodology described in Sections 3.1–3.3, we examine whether the instruments used by DH (2011) are weak, estimate the EIS, and build weak instrument robust confidence intervals for the EIS.

Next, to investigate DH’s (2011) results in the heteroscedastic case, we employ the CUE-GMM estimator along with weak identification robust tests as discussed in Sections 3.2 and 3.3. Finally, as a robustness exercise, we re-estimate the econometric specifications using Yogo’s (2004) instruments.

We use four different asset returns in order to assess which produces a more plausible estimate of the EIS.14 This also allows us to check if Yogo’s (2004) results hold for a different sample and different asset returns. Yogo’s (2004) data for the United States covers a different period (i.e., from 1947.3 to 1998.4). Second, Yogo (2004) did not use after-tax (or net) returns. Therefore, it is important to verify whether the T-Bill real net return in this new period generate the same results.

5.1. Tests for weak instruments and EIS estimates

Following Vissing-Jorgensen (2002) and DH (2011) used eight excluded instruments: two-, three-, and four-period lagged net real (or real rate if the real rate of return is used as the endogenous regressor in the specification) T-Bill rate and ND growth rate; two-period lagged bond default yield premium and bond horizon yield premium.

Table 2 presents the results of the weak instrument analysis. Focusing on the nondurables consumption specifications, the first-stage F-statistic for the SMF real returns is very low – about 2.16. As a result, the p-values for TSLS bias, Fuller-k bias, TSLS size and LIML size tests are well above 10%, indicating that SMF returns are extremely difficult to predict, leading to instruments weaker than those used by Yogo (2004). In particular, p-values above 10% for the TSLS bias and the Fuller-k bias tests indicate that these estimators are severely biased. On the other hand, the F-statistic for the T-Bill return is

<table>
<thead>
<tr>
<th>Assets</th>
<th>Average</th>
<th>Standard error</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>ND growth</td>
</tr>
<tr>
<td>T-Bill real</td>
<td>0.0041</td>
<td>0.0054</td>
<td>0.1902</td>
</tr>
<tr>
<td>SMF real</td>
<td>0.0148</td>
<td>0.0236</td>
<td>0.1867</td>
</tr>
<tr>
<td>T-Bill net real</td>
<td>0.0007</td>
<td>0.0047</td>
<td>0.2779</td>
</tr>
<tr>
<td>SMF net real</td>
<td>0.0125</td>
<td>0.0231</td>
<td>0.1901</td>
</tr>
</tbody>
</table>

Note: Quarterly asset returns are expressed as the return per quarter. ND and NDS mean ‘nondurable consumption’ and ‘nondurable plus services consumption’, respectively.

---

12 Details regarding the tax calculations can be found in Dacy and Hasanov (2011, p. 141).
13 Our descriptive analyses are not directly comparable with those of DH’s (2011) Table 1 because they used the returns in annual frequency whereas we used the returns in quarterly frequency.
14 For the sake of brevity, further results involving stock returns are omitted, but they are available upon request.
sufficiently large to reject the null of TSLS bias, but it is not large enough to reject the null of the TSLS size test. Indeed, at the 5% level, the KP test rejected the null hypothesis of underidentification for all returns. Similar weak instrument test results are found for nondurables plus services consumption. In sum, the T-Bill return can be predicted by the DH (2011) instruments whereas the SMF returns are not easily predicted.

Table 2 reports the estimates of the EIS using the TSLS, Fuller-\( k \) and LIML estimators along with DH’s (2011) instrument list. When nondurables consumption is the dependent variable, the use of the real and net real T-Bill returns do not produce statistically significant estimates of the EIS. On the other hand, when the SMF net real returns are used, the estimated EIS ranges from 0.143 to 0.385 and is different from zero at the 5% level for all three estimators. The results for the nondurables plus services are different from the previous results in the sense that the estimates using the SMF net real returns are no longer statistically significant.

Following Gomes and Paz (2011), we conduct the Sargan specification test in order to verify whether the moment conditions are valid. The Sargan test results are also reported in Table 3. As discussed earlier the Sargan test is only reliable in the absence of weak instruments, which restricts the reliable estimates to the ones using the T-Bill returns. We find that for these specifications the null hypothesis of the Sargan test is rejected, which is evidence against the specification used, and is in line with Gomes and Paz (2011) findings.

Given that weak instruments bias the point estimates of the EIS, an alternative would be to construct weak instrument robust confidence sets for the EIS by inverting the CLR test, which are reported in Table 4. For both the nondurables and nondurables plus services series, the confidence intervals for the T-Bill real and net real returns are finite but always contain zero, so we cannot rule out the case that the EIS is zero, which corroborates the results obtained so far. Indeed, in some cases the EIS point estimates based on T-Bill are negative, an unexpected result. For the SMF returns, the confidence intervals are \((-\infty, +\infty)\), i.e., uninformative for nondurables plus service consumption measure. For nondurable measure, the intervals are finite, but contains zero, which in turn weakens the conclusion that SMF delivers positive estimates of the EIS. Interestingly, in these cases there are a significant overlap between T-Bill and SMF intervals, being the latter more to the right than the former. In sum, these intervals can be interpreted as that the EIS cannot be precisely estimated.

Now, we consider the case of heteroscedastic errors that requires the use of GMM-type estimators. DH (2011) followed Vissing-Jorgensen (2002) and used iterated GMM with the assumption that the error term followed a MA(1) process. Here, we employ the CUE-GMM estimator with Newey and West (1994) HAC covariance estimator with the automatic bandwidth selection procedure.\(^{15}\) The use of the CUE-GMM is justified on the grounds of weak identification robustness as discussed in Section 3.2. Although some efficiency is lost by using Newey–West estimates of the covariance matrix, we can be assured that the consistency and the statistical significance of our results are not highly dependent on assumptions regarding the error term.

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\(^{15}\) We also employed the two-step and iterated GMM estimators; however, the estimates changed wildly depending on the estimator and the asset return used, which is evidence that weak identification is a major concern. These results are available upon request.
Table 5 reports the EIS estimates for the consumption of nondurables. The results suggest that the EIS is negative for the T-Bill returns and positive for the SMF returns. The null hypothesis that EIS equals zero of the S and JKLM tests are rejected for all specifications. The latter test results mean that the KLM test statistic is spurious; therefore, the weak identification
Table 5
Estimates of the EIS using Eq. (1) and DH's (2011) Instrument List CUE-GMM.

<table>
<thead>
<tr>
<th>Consumption Asset</th>
<th>EIS</th>
<th>S statistic</th>
<th>KLM statistic</th>
<th>JKLM statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nondurables</td>
<td>T-Bill real</td>
<td>-0.387</td>
<td>15.10* (0.088)</td>
<td>0.001 (0.999)</td>
</tr>
<tr>
<td></td>
<td>S-based CI</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>[-0.939; -0.114]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SMF real</td>
<td>0.120</td>
<td>21.43** (0.011)</td>
<td>0.720 (0.698)</td>
</tr>
<tr>
<td></td>
<td>S-based CI</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T-Bill net real</td>
<td>-0.398</td>
<td>16.74* (0.053)</td>
<td>0.114 (0.945)</td>
</tr>
<tr>
<td></td>
<td>S-based CI</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SMF net real</td>
<td>0.127</td>
<td>18.36** (0.031)</td>
<td>0.003 (0.999)</td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nondurables plus services</td>
<td>T-Bill real</td>
<td>-1.09</td>
<td>12.77 (0.173)</td>
<td>0.153 (0.926)</td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-based CI</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SMF real</td>
<td>-0.193</td>
<td>11.16 (0.265)</td>
<td>0.018 (0.991)</td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>[-1.46; -0.031]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-based CI</td>
<td>[-3.086; -0.018]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T-Bill net real</td>
<td>-0.326</td>
<td>13.98 (0.123)</td>
<td>0.00 (1.00)</td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-based Cl</td>
<td>[-2.84; -0.066]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SMF net real</td>
<td>0.067</td>
<td>26.13** (0.002)</td>
<td>0.47 (0.791)</td>
</tr>
<tr>
<td></td>
<td>KLM-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-based Cl</td>
<td>N.A.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: DH's (2011) instruments are the two-, three-, and four-period lagged net real (or real if the real rate is used) T-Bill rate and nondurables growth rate, and two-period lagged bond default yield premium and bond horizon yield premium. p-values are in parentheses. 'S-based CI' means the confidence interval obtained for the EIS by inverting the S test. 'KLM-based CI' means the confidence interval obtained for the EIS by inverting the KLM test, and it is N.A. when the K-statistic is spurious in the parameter range considered, which happens when the p-value of the JKLM is smaller than 10%. A total of 191 observations were included.

* Statistical significance at 10% level.
** Statistical significance at 5% level.

robust confidence interval based on the KLM statistic is not reliable. The weak identification robust confidence interval based on the S test is not empty only for the T-Bill real return, and suggests that the estimated EIS using T-Bill return is negative.

Also in Table 5 we report the results for nondurables plus services consumption, but now the S test only rejected the null hypothesis for the SMF net real return that presented a positive EIS point estimate. In the other cases, the null hypothesis of the S, KLM, and JKLM tests were not rejected, at the 5% level, which means that the EIS is not statistically different from zero.

The weak instrument robust confidence interval based on the S statistic is not empty only for SMF real and T-Bill net real, but in both cases it suggests that the EIS is negative. The KLM-based weak identification confidence intervals are available only for SMF real and, once again, a negative EIS is suggested.

To summarize, among these eight estimates from Table 5, the T-Bill return based EIS estimates were negative whenever it exhibited rejection of the null on the S and JKLM tests, and this result was also supported by any bounded weak instrument robust confidence interval. The SMF net real return presented a positive estimated EIS and had the null hypothesis that EIS is equal to zero rejected on both S and JKLM tests. Although on the low end of DH’s (2011) estimates, these estimates still larger than those using T-Bill returns.

5.2. Robustness checks

In this subsection, we investigate whether the preceding results are specific to DH’s (2011) instruments by re-estimating the previous specifications employing Yogo’s (2004) instruments, namely the twice-lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio.

In Table 2, we find the weak instrument tests used to verify if Yogo’s (2004) instrument list is able to predict the asset returns. For nondurables consumption, the null of underidentification of the KP test is rejected for all specifications. The SMF returns exhibited a first-stage F-statistics below 4, so the presence of weak instruments is unquestionable. As a result, the p-values for the TSLS bias and the Fuller-k bias tests are above 10%, indicating that the estimators are severely biased. The p-values for the TSLS and the LIML size tests are well above 10%. So, estimates based on these returns are unreliable. As also found by Yogo (2004), the T-Bill returns are predictable, i.e., they have F-statistics above 40. As a result, the four weak instruments tests presented p-values close to zero. Thus, Yogo’s (2004) finding that only the T-Bill return is predictable remains unchanged even when we consider net real returns. The results for nondurables plus services are very close to the estimates using the nondurables series.

Under the assumption of homoscedastic errors, Table 6 presents the estimates of the EIS using the TSLS, Fuller-k and LIML estimators along with Yogo’s (2004) instrument list. All EIS estimates for both nondurable and nondurables plus services
consumption are not statistically significant. For the specifications using the T-Bill returns, which are not plagued by weak instruments, the null hypothesis of the Sargan test was rejected at the 5% level in all specifications. Such results are in line with Gomes and Paz’s (2011) findings. As before, while the SMF delivers positive point estimates, in some cases T-Bill leads to negative values. However, differently from DH’s (2011) instrument list, now all estimates based on the SMF returns are no longer significant.

The weak instrument robust confidence intervals for the EIS based on the CLR test are calculated using Yogo’s (2004) instruments and are reported in Table 4. Contrary to the results using DH’s (2011) instruments, there are no uninformative intervals for nondurables consumption. Nonetheless, all confidence intervals contain negative and positive values, so we cannot make any conclusions regarding the sign of the EIS. The confidence intervals for nondurables plus services consumption are very similar to those using DH (2011) instruments, in the sense that only the T-Bill returns presented bounded intervals, and these intervals contain zero.

So far, applying Yogo’s (2004) approach to Dacy and Hasanov’s (2011) dataset using two different instrument sets produces weak instrument test results in accordance with Yogo (2004), and Sargan overidentification test results similar to those in Gomes and Paz (2011). Both papers used only real returns before taxes, but their main conclusions still apply to our estimates using the after-tax real returns with the exception of the estimates using the SMF net real return with nondurable consumption, which presented a positive and statistically significant EIS.

Furthermore, we found strong evidence that Yogo’s (2004) and DH’s (2011) instrument lists are unable to predict the SMF returns, but the former instrument list seems to have a slightly better performance on the weak instrument tests. This is not a surprising result since the SMF is much more volatile than the T-Bill return.

Finally, moving to the heteroskedastic error case, Table 7 reports the CUE-GMM estimates using Yogo’s (2004) instruments list. For nondurables, the null hypothesis that EIS is zero was not rejected by both the S and the JKL tests for the T-Bill returns, but the positive EIS estimates using the SMF returns presented a rejection at the 10% level in the JKL tests. The weak identification robust interval for the EIS based on the S and JKL statistic when bounded and non-empty contained zero, so we cannot discard the possibility of a negative or even zero EIS. Nevertheless, whenever the confidence intervals based on the JKL statistic exist, they are smaller than the ones based on the S test as expected.

The estimates using nondurables plus services presented a different picture. For the T-Bill real and net real returns the null hypothesis that the EIS is zero was rejected at the 10% level in the S test and at the 5% level at the JKL test, but the

---

**Table 6**

Estimates of the EIS using Eq. (1) and Yogo’s (2004) Instrument List.

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Asset</th>
<th>TSLS</th>
<th>Fuller-k</th>
<th>LIML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nondurables</td>
<td>T-Bill real</td>
<td>−0.026</td>
<td>−0.053</td>
<td>−0.057</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td>(0.141)</td>
<td>(0.142)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>8.740**</td>
<td>8.705**</td>
<td>8.705**</td>
</tr>
<tr>
<td></td>
<td>SMF real</td>
<td>0.097</td>
<td>0.129</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.107)</td>
<td>(0.116)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>7.573*</td>
<td>7.452*</td>
<td>7.445*</td>
</tr>
<tr>
<td></td>
<td>T-Bill net real</td>
<td>0.188</td>
<td>0.168</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(0.156)</td>
<td>(0.157)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>7.874**</td>
<td>7.854**</td>
<td>7.854**</td>
</tr>
<tr>
<td></td>
<td>SMF net real</td>
<td>0.104</td>
<td>0.135</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.105)</td>
<td>(0.112)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>7.343*</td>
<td>7.219*</td>
<td>7.212*</td>
</tr>
<tr>
<td>Nondurables plus services</td>
<td>T-Bill real</td>
<td>−0.058</td>
<td>−0.085</td>
<td>−0.087</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.096)</td>
<td>(0.096)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>14.381**</td>
<td>14.294**</td>
<td>14.294**</td>
</tr>
<tr>
<td></td>
<td>SMF real</td>
<td>0.066</td>
<td>0.174</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.130)</td>
<td>(0.177)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>13.589*</td>
<td>12.153**</td>
<td>12.029*</td>
</tr>
<tr>
<td></td>
<td>T-Bill net real</td>
<td>0.104</td>
<td>0.082</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.107)</td>
<td>(0.108)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>14.628**</td>
<td>14.583**</td>
<td>14.583**</td>
</tr>
<tr>
<td></td>
<td>SMF net real</td>
<td>0.072</td>
<td>0.170</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.118)</td>
<td>(0.149)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sargan statistic</td>
<td>13.273**</td>
<td>11.952**</td>
<td>11.865**</td>
</tr>
</tbody>
</table>

**Note:** Yogo’s (2004) instruments are the twice-lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio. Standard errors are in parentheses. A total of 191 observations were included.

**Statistical significance at 5% level.**

**Statistical significance at 10% level.**

---

16 We also used two-step and iterated GMM to estimate the EIS. As before, weak identification leads to disparate estimates. These results are available upon request.
EIS estimate for the T-Bill real return was negative, and the S-test based weak identification robust interval included zero in both cases. For the SMF real and net real returns the null of EIS equal to zero was rejected at the 5% level for both the S and JKLM tests, and these estimates were positive but had a small magnitude of about 0.06.

6. Conclusions

The Elasticity of Intertemporal Substitution (EIS) is one of the key parameters in the Economics and Finance literature whose estimation is generally done by means of the representative consumer Euler Equation, which is derived assuming that the representative agent has CRRA utility and the asset returns considered are the T-Bill or the S&P 500 portfolio. The EIS estimates are obtained using instrumental variables, which require valid and relevant instruments.

Yogo (2004), among others, estimated EIS to be close to zero, but one of the reasons for this puzzling result is the use of weak instruments. On the other hand, Dacy and Hasanov (2011) argued that the common practice in literature of using a single asset return (before taxes) to measure the representative agent portfolio return is inappropriate. Accordingly, they built an aggregate asset, the synthetic mutual fund (SMF), using the observed shares of each asset on an average household portfolio. Their EIS estimates using the SMF return are approximately 0.20 and statistically different from zero.

In this paper, we carefully investigated if estimates based on T-bill and SMF returns are plagued by weak instruments and found that only the T-Bill return specifications are less affected by the weak instrument problem. Nevertheless, for the T-Bill return the EIS estimates are either negative or positive and small in magnitude. Furthermore, the weak instrument robust confidence intervals contain zero, indicating that EIS may not be different from zero. Most of the EIS estimates using the SMF returns are positive and some are statistically significant, being in the lower end of DH’s (2011) estimates. Thus in some cases, the SMF net real return performed better than the T-Bill returns. Our results corroborate Yogo’s (2004) findings that the T-Bill returns do not suffer from weak instruments and lead to close to zero EIS estimates, which could indicate that it is not the asset return considered by the average household. At the end of the day, the final word on the true estimate of the EIS is still to come.

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