Robust monetary policy with the consumption-wealth channel

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This paper studies how the central bank's concerns about model uncertainty affect the design of monetary policy in the presence of wealth effects. If all exogenous disturbances are white noises, increasing the preference for robustness or the size of wealth effects implies more aggressive policy responses to cost shocks. Under persistent shocks, numerical simulations show that increasing the preference for robustness continues to imply more aggressive responses to cost shocks. By contrast, stronger wealth effects lead to less aggressive responses, dampening the effect of model uncertainty on interest rate dynamics.

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1. Introduction

Uncertainty about the most plausible model for the economy leads monetary authorities to design policies that aim to be effective even in worst-case scenarios, in which the model adopted by the central bank is no longer valid. Robust monetary policy is designed to avoid poor economic outcomes in the presence of misspecified models.

The literature on robust monetary policy is growing rapidly, especially the branch advanced by Hansen and Sargent (2008), which is related to the use of control theory. Giordani and Söderlind (2004) develop numerical methods to compute optimal robust policies. Leitemo and Söderström (2008a, 2008b) study the robust optimal monetary policy in the new Keynesian framework for closed and open economies, and obtained closed-form solutions under the assumption of white noise shocks. Tillmann (2009) examines the robust monetary policy with the cost channel of monetary transmission.

Robust policies depend on a reference model, i.e., a model that the policymaker believes to be the most likely description of an economic system. The reference model features plausible channels of monetary policy transmission. One potentially important transmission mechanism is the consumption-wealth channel, which allows asset prices to affect real activity.

The views on the importance of asset prices for consumption vary widely. For instance, Ludvigson and Steindel (1999) argue that wealth effects on consumption in the U.S. are unstable and measured with a great deal of uncertainty. In addition, Ludvigson et al. (2002) show that the consumption-wealth channel plays a minor role in the propagation of...
monetary policy in the U.S. On the other hand, Bertaut (2002) and Ludwig and Sløk (2004) point to significant impacts of equity and housing prices on consumption, especially in economies with market-based financial systems. Altiìsimo et al. (2005) survey the empirical literature on wealth effects and conclude that, in general, empirical studies reveal a statistically significant relationship between wealth and consumption.

In summary, the empirical evidence suggests, though not unanimously, that the effect of wealth on consumption is important. Moreover, its size seems to vary over time in a given country, as documented by Ludvigson and Steindel (1999), and across economies, as shown by Bertaut (2002) and Ludwig and Sløk (2004).

In addition to the empirical relevance of the consumption-wealth channel, central banks, concerned with its macroeconomic effects, incorporate the consumption-wealth channel in their view of the monetary transmission mechanism. For instance, the European Central Bank (ECB) includes the consumption-wealth channel in the description of the monetary transmission mechanism in Europe, according to its web site and the ECB Monthly Bulletin of July 2000.

Moreover, some central banks and policy institutions incorporate the consumption-wealth channel in models used by their staff members in policy analysis. Brayton and Tinsley (1996) show that the FRB/US includes a direct effect of asset prices on aggregate consumption. The NONAME model of the National Bank of Belgium, described in Jeanfils and Burggraeve (2008), embodies overlapping generations of consumers, generating an aggregate consumption function displaying the consumption-wealth channel. Additionally, models based on overlapping generations, featuring the consumption-wealth channel, have been developed to guide policy discussions at the Bank of England and the IMF. Harrison et al. (2005) describe the Bank of England Quarterly Model (BEQM) and Kuhmof et al. (2010) present the IMF Global Integrated Monetary and Fiscal Model (GIMF). In both models, households are modeled in a way similar to the structure described in Appendix of this paper. In short, I show evidence that the consumption-wealth channel is a feature that actual central banks incorporate in their view of the economy’s structure.

This paper studies how central banks’ concerns about model uncertainty affect the design of monetary policy in the Blanchard–Yaari framework, which is an overlapping generations model featuring the consumption-wealth channel. While central banks plausibly consider the consumption-wealth channel in their models, the reasons why they might use the Blanchard–Yaari framework as their reference model are not so evident. In the following paragraphs, I argue that the Blanchard–Yaari model has merits to serve as an empirically more relevant reference model.

Indeed, the Blanchard–Yaari model provides a more empirically plausible specification for the Phillips curve. Kuttner and Robinson (2010) survey the literature on the new Keynesian Phillips curve. They show evidence that the Calvo parameter in the Phillips curve based on the representative agent model is too high in comparison with estimates from microeconomic studies. In fact, the slope of the Phillips curve has become flatter over time, implying even higher values for the Calvo parameter. Though there are alternative models that may be able to generate Calvo parameter estimates in line with the micro evidence, thus explaining the flattening of the Phillips curve, the Blanchard–Yaari model offers a reasonable explanation based on demographic changes.

First, the Blanchard–Yaari model can generate a given value of the slope of the Phillips curve with a smaller Calvo parameter, which is more in line with the micro evidence compared to the representative agent case. In fact, a given value of the slope is compatible with a smaller Calvo parameter because the slope is a positive function of the size of the wealth effect in the Blanchard–Yaari Phillips curve. The positive size of the wealth effect, due to a positive probability of dying and a positive real wealth to output ratio in steady state, compensates the negative effect of a smaller Calvo parameter on the slope. Second, given the Calvo parameter, small values of the slope over time are compatible with a decreasing probability of dying over time, which is in line with the empirical evidence on population ageing documented in Bloom and Canning (2006).

In addition, the asset meltdown hypothesis can reduce the size of the wealth effect through a reduction in steady state asset prices, leading to small values of the slope. This hypothesis, discussed first in Poterba (2001) and Abel (2001), states that a decreasing asset demand caused by smaller working-age cohorts and an increasing asset supply due to retired workers lead to falling asset prices. Despite the mixed empirical support for the asset meltdown hypothesis discussed in Bosworth (2004), Davis and Li (2003) and Takáts (2012) provide sound evidence in defense of this hypothesis. These demographic changes, accounted for by the Blanchard–Yaari model, offer a plausible interpretation of the flattening of the Phillips curve documented in Kuttner and Robinson (2010).

A more general question is whether an empirical version of the Blanchard–Yaari model, as described in Nisticó (2012), fits the data better than the standard representative agent model. Dynamic stochastic general equilibrium (DSGE) modelers have embedded the perpetual-youth story in empirical structural models with nominal rigidities and backward-looking dynamics. Castelnuovo and Nisticó (2010) estimate a closed-economy model for the U.S., showing that the consumption-wealth channel is empirically important for the monetary transmission mechanism. In addition, they perform a likelihood-based comparison with the representative agent model and obtain results that indicate the superiority of the Blanchard–Yaari model as a more plausible specification. Milani (2011) and Funke et al. (2011) provide analogous evidence of the empirical relevance of the consumption-wealth channel in a two-country model and in a small open economy setting, respectively.

In addition, the Blanchard–Yaari framework has been used to address important issues in monetary economics. Piergallini (2006) studies optimal monetary policy in a model with real balance effects. Annicchiarico et al. (2008) study
the interaction between fiscal and monetary policy in a variant of Piergallini (2006). Finally, Nisticò (2012) studies the role of monetary policy for stock-price dynamics.

In contrast to these previous applications of the Blanchard–Yaari model, I focus on how changes in the preference for robustness and in the size of wealth effects affect the degree of aggressiveness of optimal monetary policy in responding to cost shocks and to shocks to the natural level of output. The following paragraphs summarize the main results.

Assuming white noise disturbances, I find the analytical solution to the robust control problem. According to the solution, an increase in the preference for robustness implies more aggressive policy responses to cost shocks but not to shocks to the natural level of output. This is in line with the result in Leitemo and Söderström (2008a). In addition, the output gap, interest rate and equity prices become more sensitive to cost shocks. I also study how the size of wealth effects affects the aggressiveness result. I find that monetary policy responses to cost shocks are even more aggressive if the consumption-wealth channel has an important role in the transmission mechanism.

Additional analytical results show that the monetary policy problem in the Blanchard–Yaari model is isomorphic to the same problem in a simple new Keynesian closed economy characterized by an aggregate demand equation based on logarithmic preferences, a more volatile Phillips curve disturbance, and a larger output gap-inflation tradeoff. In this paper, the output gap-inflation tradeoff measures the loss or gain in output necessary to bring inflation back to its target, according to the optimal monetary policy.

All the results above depend crucially on the assumption of white noise shocks. For the case of persistent shocks, I use numerical methods to solve the model. According to the numerical solution, an increase in the preference for robustness also leads to more aggressive responses to cost shocks. In contrast, stronger wealth effects are associated with less aggressive responses, attenuating the effect of model uncertainty on interest rates. This last finding depends on the importance of output stabilization for the central bank. As the importance of wealth effects increases, the central bank may be forced to reduce interest rates to minimize the impact of a plunge in equity prices on aggregate demand caused by cost shocks.

The paper is organized in three additional sections. In Section 2, I present a version of the Blanchard–Yaari model developed by Nisticò (2012). In Section 3, I derive a closed-form solution to the optimal robust policy under discretion and present numerical results concerning the case of persistent shocks. Section 4 concludes and provides additional discussion of the results.

2. The model

In this section, I present the log-linear approximation of the sticky price overlapping generation (perpetual youth version) economy developed by Nisticò (2012), which features a consumption-wealth channel by which asset prices can have a direct effect on aggregate demand. An Appendix provides more details of the model, which will serve as the reference model in Section 3.

The following equations define a linear rational expectations model, approximately describing the equilibrium conditions.

\[ q_t = \beta E_t (q_{t+1}) + (1 - \beta) \left[ 1 + \frac{1 + \chi}{1 - \mu} \right] E_t (x_{t+1}) - [r_t - E_t (\pi_{t+1})] + (1 - \tilde{\beta}) E_t (y^n_{t+1}) + \left[ \frac{(1 - \tilde{\beta})}{(1 - \mu)} \right] E_t (u_{t+1}) \]  

\[ x_t = \frac{\psi}{1 + \psi} q_t + \frac{1}{1 + \psi} E_t (x_{t+1}) - \frac{1}{1 + \psi} [r_t - E_t (\pi_{t+1})] + \frac{1}{1 + \psi} E_t (y^n_{t+1}) - y^n_t \]  

\[ \pi_t = \tilde{\beta} E_t (\pi_{t+1}) + \frac{(1 - \phi)(1 - \tilde{\beta} \phi)}{\phi} (1 + \chi) x_t + \frac{(1 - \phi)(1 - \tilde{\beta} \phi)}{\phi} u_t \]

The endogenous variables are inflation (\(\pi_t\)), the output gap (\(x_t\)), real stock prices (\(q_t\)) and the interest rate (\(r_t\)). All variables are measured in log deviations from their steady-state values. The exogenous shocks are the natural level of output (\(y^n_t\)) and the cost shock (\(u_t\)). The first equation summarizes the dynamics of real stock prices; the second describes the aggregate demand; and the third is a new Keynesian Phillips curve characterizing inflation dynamics.

The basic parameters are the subjective discount factor (\(\beta\)), the probability of death of an individual in the perpetual youth model (\(\gamma\)), the Calvo parameter (\(\phi\)) measuring the degree of price stickiness and the price elasticity of demand for each intermediate good in the production of the final good (\(\nu\)).

The auxiliary parameters are

\[ \phi = 1 - \beta (1 - \gamma), \quad \psi = \frac{\beta \gamma \phi}{1 - \phi} \left( \frac{Q + D}{Y} \right), \quad \tilde{\beta} = \frac{\beta}{(1 + \psi)}, \quad \mu = \frac{\nu}{\nu - 1} \quad \text{and} \quad \chi = \frac{L}{1 - L}. \]

The variables \(Q, D, Y\) and \(L\) denote steady-state levels for real asset prices, dividends, output and labor, respectively.

3. The robust monetary policy

Uncertainty about the most plausible model for the economy has stimulated a body of research trying to characterize desirable policies in the presence of model uncertainty. The robust approach to monetary policy assumes that the central
bank has a model believed to be a good description of reality, but is averse to uncertainty. If reality deviates from the model in a way hard to be described by probabilistic statements, the policymaker wants to avoid poor performances. Thus, the monetary policy design problem should take into account the effects of specification errors and try to be robust against them. Robust policies are therefore designed to avoid poor economic outcomes in worst-case situations.

The central bank, under the robust control approach, minimizes a loss function, considering a reference model which is a reasonable representation of the law of motion of the economy. The loss function is assumed to be quadratic and the model linear. The central bank realizes that the true description of the economy may deviate from the reference model. However, it cannot specify a probability distribution associated with these deviations. To model such highly unstructured uncertainty, shock terms representing model misspecifications are appended to the equations defining the linear model. These disturbances are controlled by a fictitious evil agent.

The initial minimization problem, with the introduction of the evil agent, becomes a min–max problem. Since the evil agent is just a way to model the planner’s precautionary behavior, it knows the planner’s reference model and loss function, which it wants to maximize. According to its preference for robustness, the central bank specifies a tolerance level for the magnitude of misspecifications, defining, in the neighborhood of the reference model, a set of alternative models that the central bank cares about.

In mathematical terms, using the notation in Giordani and Söderlind (2004), the problem can be formulated as follows.

\[
\begin{align*}
\text{Min} & \quad \text{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2] \\
\text{s.t.} & \quad q_t = \beta E_t(q_{t+1}) + (1-\beta)\gamma_1E_t(x_{t+1}) - [\gamma_1 E_t(\pi_{t+1})] + (1-\beta)\rho_u y_t^u + (1-\beta)\gamma_2 \rho_n u_t \\
& \quad x_t = (1-\gamma_2)q_t + \gamma_2 E_t(x_{t+1}) - \gamma_2 [\gamma_1 E_t(\pi_{t+1})] - (1-\gamma_2 \rho_n) y_t^u \\
& \quad \pi_t = \beta E_t(\pi_{t+1}) + \gamma_3 x_t + \gamma_4 u_t \\
& \quad y_t^u = \rho_n x_{t-1} + \gamma_5 (e_t^u + v_t^u) \\
& \quad u_t = \rho_u u_{t-1} + \gamma_4 (e_t^u + v_t^u) \\
& \quad E_0 \sum_{t=0}^{\infty} \beta^t [(v_t^u)^2 + (v_t^u)^2] \leq \eta
\end{align*}
\]

where \( \gamma_1 = 1 + (1+\chi)/(1-\mu) \), \( \gamma_2 = 1/(1+\psi) \), \( 1-\gamma_2 = \psi/(1+\psi) \), \( \gamma_3 = ((1-\phi)(1-\phi\beta)/\phi)(1+\chi) \), \( \gamma_4 = (1-\phi)(1-\phi\beta)/\phi \) and \( \gamma_5 = (1-\beta)/(1-\mu) \).

I assume that the central bank sets the short-term interest rate \( r_t \) to minimize the standard quadratic loss function \( E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2] \). The overlapping generations structure introduces additional difficulties for the specification of a utility-based welfare criterion. In fact, it is difficult to aggregate preferences in the Blanchard–Yaari set-up. Therefore, contrary to Woodford (2003) and Walsh (2003), the quadratic loss function cannot be interpreted as a second-order approximation of the utility function of a representative agent.

In spite of its ad hoc nature, there are advantages to adopting the standard quadratic loss function First, this simple specification allows a closed-form solution to the robust monetary policy problem. Second, the results can be easily compared to the findings in Leitemo and Söderström (2008a), who use the basic new Keynesian framework as the reference model.

The first five constraints describe the reference model with the presence of the evil agent. The variables \( e_t^u \) and \( e_t^n \) are zero mean iid shocks; \( v_t^u \) and \( v_t^u \) are the evil agent’s control variables. Finally, \( \rho_n \) and \( \rho_u \) are the persistence parameters for the structural shocks \( y_t^u \) and \( u_t \) in the reference model, which follow an AR(1) specification.

I have to rewrite the cost shock and the shock to the natural level of output in such a way that \( e_t^u \) and \( e_t^n \) have unit variance. The evil agent’s control variables are multiplied by the variances of structural shocks, so random noises are masking to some extent the true law of motion of the economy.

In addition, the last constraint is sometimes referred to as the evil agent’s budget constraint. Under the evil agent metaphor, the central bank is able to specify a certain amount of noise that this agent can use in order to perturb the reference model. The parameter \( \eta \) represents the amount of misspecification tolerated by the central bank. The standard dynamic control problem under rational expectations sets \( \eta = 0 \), implying \( v_t^u = v_t^u = 0 \).

The equilibrium is found by combining the linear law of motion with policy functions for \( r_t \), \( v_t^u \) and \( v_t^u \), which are the solution of the min-max problem described above. The literature has focused on two situations: the worst-case model and the approximating model. The worst-case model defines an equilibrium configuration characterized by the presence of the evil agent, against which the monetary policy response is designed. The approximating model solution uses policy functions from the worst-case model when there is no evil agent.
3.1. The analytical solution

I assume that neither the central bank nor the evil agent has access to a commitment technology. Therefore, expectations are given and monetary policy is set under discretion. To find closed-form solutions, following Leitemo and Söderström (2008a, 2008b), I assume that all shocks are white noises, i.e., $\rho_n = \rho_u = 0$.

3.1.1. The worst-case model

The optimality conditions lead to the following results.

Proposition 1. With cost shocks, there exists a tradeoff between inflation and the output gap variability. This tradeoff is not affected by the robustness parameter $\theta$.

Proof. The first optimality condition is $x_t = - (x_3 / \lambda) \pi_t$. The Lagrange multiplier on $E_0 \sum_{s=0}^{\infty} \beta^s [(v_s^u)^2 + (v_s^p)^2] < \eta$ is denoted by $\theta$, known as the robustness parameter, which is inversely related to $\eta$. Therefore, a decrease in $\theta$ means an increase in the preference for robustness. The first optimality condition describes a targeting rule for the central bank. This rule is a “lean against the wind” policy that instructs the central bank to contract economic activity below its potential level when inflation is above the target; and vice versa when it is below the target. The output gap-inflation tradeoff is the coefficient $x_3 / \lambda$, which controls the amount of output loss or gain that is necessary to bring inflation back to its target. The proposition follows directly from the first optimality condition, since $\theta$ does not appear in this equation. \qed

Proposition 2. Since cost shocks introduce a tradeoff between inflation and the output gap, the evil agent adds noise to these shocks. The amount of noise is an increasing function of inflation, the standard deviation of the Phillips curve disturbance ($\sigma_u$) and the preference for robustness ($1/\theta$).

Proof. The second optimality condition is $v_t^u = (\sigma_u \theta / \pi_t)$ and the Phillips curve disturbance is $\zeta_t = \sigma_u u_t$. The proposition follows directly from these equations. \qed

Proposition 3. Since shocks to the natural level of output do not introduce any tradeoff between inflation and the output gap, the evil agent does not add any noise to these shocks.

Proof. The third optimality condition is $v_t^u = 0$. To show this claim, just inspect this optimality condition. \qed

Since the central bank is able to offset the effects of shocks to the natural level of output and there is no persistence, the only relevant variable that can be used to add uncertainty to the system is $u_t$.

The importance of cost shocks in generating a tradeoff between inflation and the output gap variability. This tradeoff is not affected by the size of the wealth effect. In addition, inflation and shocks to the natural level of output enter the interest rate equation. The reaction of the interest rate to inflation depends on the inflation-output gap tradeoff $x_3 / \lambda$. The reaction to shocks to the natural level of output is set to neutralize their effects on the economy.
Proof. Imposing $\nu_i^t = 0$ and setting expectations to zero in (1) and (2), the interest rate and equity prices are

$$\ r_t = \left(1-\gamma_2\right) \frac{x_t}{\gamma_2} q_t - \sigma_n e_t^n (1-\gamma_2) \frac{x_t}{\gamma_2} q_t + \frac{\gamma_3}{\lambda} \pi_t - \sigma_n e_t^n
$$

Using $q_t = -r_t$ from (1), I obtain expressions written in terms of the underlying shocks:

$$\ r_t = - \frac{\sigma_n (1+\gamma) x_t}{\lambda + \gamma^2 A_1} e_t^n - \sigma_n e_t^n \quad (6)
$$

$$\ q_t = -r_t = - \frac{\sigma_n (1+\gamma) x_t}{\lambda + \gamma^2 A_1} e_t^n + \sigma_n e_t^n \quad (7)
$$

Proposition 6. For small degrees of misspecification, the central bank responds to cost shocks by tightening monetary policy. Since $|\partial r_t / \partial x_t^n| = |\partial q_t / \partial x_t^n| = |\partial x_t / \partial x_t^n|$ implies $\partial q_t / \partial r_t = \partial x_t / \partial r_t = -1$, monetary policy responses to cost shocks have a negative full impact on the output gap and equity prices.

Proof. To show this, just take the limits as $\theta$ goes to infinity of the derivatives of $x_t$, $r_t$ and $q_t$ with respect to $e_t^n$ in Eqs. (5)–(7).

3.1.2. The approximating model

The worst-case model discussed so far represents a situation in which the central bank’s worst fears about model uncertainty come true. In the approximating model, the policy decision and expectations are in line with the central bank’s fears about misspecification. However, in practice misspecification is absent and the evil agent sets his control variables to zero. The following proposition summarizes important features of the equilibrium.

Proposition 7. The approximating model solution has no impact on the sensitivity of the output gap and equity prices to cost shocks and to shocks to the natural level of output, but changes inflation behavior since the noise added by the evil agent is absent.

Proof. The solution combines Eq. (6) and the original linear rational expectations model without the evil agent’s control variables, described by Eqs. (1)–(3). The interest rate equation is the same as in the worst-case model and the expressions for the output gap and equity prices remain unchanged. Thus, Eqs. (5) and (7) also describe the behavior of the output gap and equity prices under the approximating model. Using Eq. (5) in the Phillips curve (3) and setting $E_i(\pi_{t+1}) = 0$ yields $\pi_t = [\sigma_n - \sigma_u (1+\gamma) x_t^2 / (\lambda + \gamma^2 A_1)] \sigma_n e_t^n$. Therefore, inflation dynamics follow a different equation in the approximating model. $\square$

3.1.3. An isomorphic monetary policy problem

Proposition 8, which resembles the findings in Clarida et al. (2001) for a small open economy, states that the Blanchard–Yaari model can be interpreted as a representative agent model with special characteristics.

Proposition 8. The monetary policy problem in the presence of wealth effects is isomorphic to the same problem in a simple representative agent new Keynesian closed economy with an aggregate demand equation implied by logarithmic preferences, a more volatile Phillips curve disturbance and a larger output gap–inflation tradeoff.

Proof. I inspect the optimality conditions and compare them to the ones obtained by Leitemo and Söderström (2008a) to conclude that the robust monetary policy in the presence of wealth effects is isomorphic to the one related to a new Keynesian closed economy model, characterized by the following reference model:

$$\ x_t = E_i(x_{t+1}) - [r_t - E_i(\pi_{t+1})] - \sigma_n e_t^n
$$

$$\ \pi_t = \bar{\beta} E_i(\pi_{t+1}) + \gamma x_t + \gamma_4 \sigma_n e_t^n
$$

The first equation is the aggregate demand equation associated with a representative agent economy with logarithmic preferences. This equation is also known as the IS curve. In fact, with white noise disturbances, expectations are irrelevant for computation of the equilibrium. In this context, an increase in the interest rate has two effects on aggregate demand. The first, given by $-\gamma_2 r_t$, is a direct effect. The second, given by $- (1-\gamma_2) r_t$, comes from the consumption-wealth channel, accounting for the fact that $q_t = - r_t$. The specification of the first equation takes into account both effects.

The second equation is a new Keynesian Phillips curve. The Phillips curve disturbance is $\zeta_t = \gamma_4 \sigma_n e_t^n$ with volatility (the variance of $\zeta_t$) given by $\gamma_4^2 \sigma_n^2$, which depends on $\gamma_4$. The output gap–inflation tradeoff is the coefficient $\gamma_3 / \lambda$ in the first optimality condition given by $x_t = - (\gamma / \lambda) \pi_t$. This coefficient measures the loss or gain in output, prescribed by the targeting rule given by the first optimality condition, which is necessary to bring inflation back to its target. The output gap–inflation tradeoff is a function of the Phillips curve slope coefficient $\gamma_3$.

The Blanchard–Yaari model ($\gamma > 0$) has a more volatile Phillips curve disturbance and a larger output gap–inflation tradeoff than the representative agent model ($\gamma = 0$). Since $\gamma_3$ and $\gamma_4$ depend on $\gamma$, to establish these facts, I need to show that $\gamma_3$ and $\gamma_4$ are increasing in $\gamma$. 


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First, consider the following expressions for the size of the wealth effect on the Euler equation (SWE) and for $\psi$:

$$SWE = \frac{\gamma}{(1-\gamma)} = \frac{\gamma}{1-\gamma} - \beta_l^\gamma$$

$$\psi = \frac{\beta_l^\gamma \varphi}{1-\varphi} \left(\frac{Q+D}{Y}\right) = \beta_l^\gamma \left(\frac{Q+D}{Y}\right) \left(\frac{\gamma}{\beta_l(1-\gamma)}\right)$$

Combine the above equations to obtain $\psi = \beta_l^2 \left(\frac{Q+D}{Y}\right)^{SWE}$

Since $0 < \beta < 1$ and $0 < \gamma < 1$ and assuming positive real wealth $(Q+D)$ in the steady state, I have the following derivatives:

$$\frac{\partial SWE}{\partial \gamma} = \frac{1-\beta(1-\gamma)^2}{(1-\gamma)^2} > 0 \quad \text{and} \quad \frac{\partial \psi}{\partial \gamma} = \beta_l^2 \left(\frac{Q+D}{Y}\right) dSWE/d\gamma > 0$$

Second, consider

$$\tilde{\beta} = \frac{1}{1+\psi}, \quad \tilde{\phi} = (1-\phi)(1-\phi \tilde{\beta})$$

The derivatives of $\tilde{\beta}$, $\tilde{\phi}$, and $\tilde{\psi}$ with respect to $\gamma$ are

$$\frac{\partial \tilde{\beta}}{\partial \gamma} = \frac{\beta_l}{(1+\psi)^2} \frac{d\psi}{d\gamma} < 0, \quad \frac{\partial \tilde{\phi}}{\partial \gamma} = -(1+\phi)(1-\phi) \frac{\partial \tilde{\beta}}{\partial \gamma} > 0 \quad \text{and} \quad \frac{\partial \tilde{\psi}}{\partial \gamma} = -(1-\phi) \frac{\partial \tilde{\psi}}{\partial \gamma} > 0$$

The volatility of $\zeta_4$ depends on $x_4$ and the tradeoff between inflation and the output gap is a function of $x_3$. Since $x_3$ and $x_4$ are increasing in $\gamma$, they are consequently bigger than their counterparts in the representative agent case in which $\gamma = 0$. \qed

To emphasize the differences between the Blanchard–Yara model and the representative agent model, I compute expressions for the ratio between Phillips curve disturbance variances in both models, as well as the output gap-inflation tradeoff ratio. I denote the variance of the Phillips curve disturbance in the Blanchard–Yara and in the representative agent model as $\text{var}(\zeta_4^{BY})$ and $\text{var}(\zeta_4^{RA})$, respectively. I use the expression for $\var(\zeta_4)$ to compute the variance ratio $\frac{\text{var}(\zeta_4^{BY})}{\text{var}(\zeta_4^{RA})} = (1+\phi/(1+\psi))(\phi \beta/(1-\phi \beta))^2$, which is clearly greater than one. The output gap-inflation tradeoff ratio and the Phillips curve slope coefficient ratio are both equal to $1 + (\psi/(1+\psi))(\phi \beta/(1-\phi \beta)).$

3.2. Further analytical results

3.2.1. The effects of robustness

The following results summarize the sensitivity of the macroeconomic variables to $\theta$.

**Proposition 9.** An increased preference for robustness makes the output gap, interest rate and equity prices more sensitive to cost shocks and more volatile. The increased volatility due to cost shocks is the same in the worst-case model and in the approximating model.

**Proof.** The expressions for the output gap, interest rate and equity prices are the same in the worst-case model and in the approximating model. They take the form $\pi_t = -A_2 \pi_t^\gamma$, $r_t = A_2 \pi_t^\gamma - \sigma_t v_n^\gamma$ and $q_t = -A_2 \pi_t^\gamma + \sigma_t v_n^\gamma$, where $A_2 = \sigma_u(1+\chi)\pi_2^\gamma / (\lambda + \pi_2^\gamma A_1)$ and $\lambda_1 = (1+\gamma)^2/\lambda / \theta$. Since $x_4$ does not depend on $\theta$, $\partial A_2 / \partial \theta = (\partial A_2 / \partial A_1)(\partial A_1 / \partial \theta) = -(\sigma_u(1+\chi)\pi_2^2 / (\lambda + \pi_2^2 A_1)^2) (\pi_2^2 / \theta^2) < 0$. \qed

**Proposition 10.** In the worst-case model, an increased preference for robustness makes inflation more sensitive to cost shocks and more volatile.

**Proof.** Inflation, in the worst-case model, is given by $\pi_t = A_2 \pi_t^\gamma$, where $A_3 = \lambda \sigma_u x_4 / (\lambda + \pi_2^2 A_1)$. Since $x_4$ does not depend on $\theta$, $\partial A_3 / \partial \theta = (\partial A_3 / \partial A_1)(\partial A_1 / \partial \theta) = -(\lambda \sigma_u x_4 / (\lambda + \pi_2^2 A_1)^2) (\pi_2^2 / \theta^2) < 0$. \qed

**Proposition 11.** In the approximating model, an increased preference for robustness makes inflation less sensitive to cost shocks and less volatile.

**Proof.** Inflation, in the approximating model, takes the form $\pi_t = A_4 \pi_t^\gamma$, where $A_4 = \sigma_u x_4 - \sigma_u(1+\chi)\pi_2^2 / (\lambda + \pi_2^2 A_1)$. The derivative of the coefficients $A_4$ is $\partial A_4 / \partial \theta = (\partial A_4 / \partial A_1)(\partial A_1 / \partial \theta) = (\sigma_u(1+\chi)\pi_2^2 / (\lambda + \pi_2^2 A_1)^2) (\pi_2^2 / \theta^2) > 0$. \qed

These results are all in line with the findings in Leitemo and Söderström (2008a). Due to Proposition 8, the model with wealth effects displays the same behavior as the basic new Keynesian model. The differences between Propositions 10 and 11 are due to the action of the evil agent. In the worst-case model, the evil agent adds more noise. Thus, inflation is more volatile.
3.2.2. The role of wealth effects

By inspecting Eqs. (1)–(3), it is easy to see that the parameter \( \psi \) is governing the impact of equity prices on the economy. The parameter \( \psi \) measures the total size of the wealth effect in the linear rational expectations system. In fact, \( \psi \) combines the direct strength of the wealth effect on consumption (SWE) with the financial wealth to GDP ratio in steady state, which can be interpreted as a measure of the size of the stock market wealth. Therefore, market-based financial systems tend to have high values for \( \psi \), as suggested by some empirical studies discussed in the introduction.

**Proposition 8** shows that \( z_4 \) is an increasing function in the total size of the wealth effect. In addition, the coefficient \( A_4 \) does not depend on \( z_4 \). For this reason, to assess the effect of an increase in the strength of the wealth effect, it is sufficient to study how \( A_2 \), \( A_3 \) and \( A_4 \) change when \( z_4 \) increases or decreases. The following propositions summarize the sensitivity of these coefficients to \( z_4 \).

**Proposition 12.** A stronger wealth effect makes the output gap, interest rate and equity prices more sensitive to cost shocks; therefore, these variables are more volatile. The increased volatility due to cost shocks is the same in the worst-case model and in the approximating model.

**Proof.** The expressions for the output gap, interest rate and equity prices are the same in the worst-case model and in the approximating model. They take the form \( x_t = -A_3 \pi_{t0} + \tau_t = A_2 \pi_{t0} - \sigma_w \pi_{t0} \) and \( q_{t0} = -A_2 \pi_{t0} + \sigma_w \pi_{t0} \), where \( A_2 = \sigma_1 (1 + \chi) \xi_2 / (\lambda + \xi_2^2 A_1) \) and \( A_1 = (1 + \chi)^2 - \lambda / \theta \). The derivative of \( A_2 \) with respect to \( z_4 \) is \( \partial A_2 / \partial z_4 = 2 \sigma_1 (1 + \chi) \xi_4 \lambda / (\lambda + \xi_2^2 A_1)^2 > 0 \). \( \square \)

**Proposition 13.** The impact of a stronger wealth effect on inflation depends on particular parameter values in the worst-case model and in the approximating model.

**Proof.** Inflation, in the worst-case model, is given by \( \pi_t = A_3 \pi_{t0} \), where \( A_3 = \lambda \sigma_n \xi_4 / (\lambda + \xi_2^2 A_1) \). In the approximating model, inflation takes the form \( \pi_t = A_4 \pi_{t0} \), where \( A_4 = \sigma_4 \lambda - \sigma_n (1 + \chi) \xi_2^2 / (\lambda + \xi_2^2 A_1) \). The derivatives for the coefficients \( A_3 \) and \( A_4 \) are \( \partial A_3 / \partial z_4 = \lambda \sigma_n (\lambda - \lambda \xi_2^2 A_1) / (\lambda + \xi_2^2 A_1)^2 \) and \( \partial A_4 / \partial z_4 = (\lambda^2 + \xi_2^2 A_1^2) / (\lambda + \xi_2^2 A_1)^2 \). The sign of both derivatives is ambiguous. For a given amount of preference for robustness, \( \partial A_3 / \partial z_4 \) depends on the relative importance of output stabilization vis-à-vis the volatility of the Phillips curve disturbance, which is proportional to \( \xi_2^2 \). The derivative \( \partial A_4 / \partial z_4 \) depends on the volatility of the Phillips curve disturbance and the slope of the Phillips curve, since \( (1 + \chi)^2 = (\alpha^2 / \xi_2^2) \). \( \square \)

To understand these results, note that equity prices affect aggregate demand and the Phillips curve. In fact, firms use the aggregate stochastic discount factor, \( A_{t,t+k} \), in order to discount future profits. As a result, since \( A_{t,t+k} \) is an average over agents which gives more importance to newborns with no wealth and high marginal utility of consumption, shocks to marginal costs today have a greater impact compared to the representative agent case. This discounting behavior affects the slope of the Phillips curve as well as the volatility of the Phillips curve disturbance.

The central bank tends to act more aggressively because the effects of cost shocks are short-lived. Thus, the output gap and equity prices become more sensitive to cost shocks. The behavior of inflation, on the other hand, depends on the relative strength of demand and supply repercussions caused by changes in the size of the wealth effect.

3.3. The case of persistent shocks

Now I extend the analysis to the case of persistent shocks, using the numerical methods outlined in Giordani and Söderlind (2004). In what follows, I study a situation with persistent shocks (\( \rho_w = \rho_r = 0.9 \)). I use the same degree of persistence for both disturbances. I also set their variances to 1 (\( \sigma_w = \sigma_r = 1 \)). The results change quantitatively as a function of persistence, but their qualitative features remain the same.

I calibrate the artificial economy using standard parameters for the U.S. economy. The values for the structural parameters come from the baseline parameterization in Söderström et al. (2005). I set \( \beta = 0.99, \phi = 0.75, \mu = 1.2, \chi = 0.5 \) and \( \lambda = 0.1 \). The range of values for \( \psi \) is chosen according to plausible values for the steady-state gross nominal interest rate, given by \( R = (1 + \psi) / \beta \).

3.3.1. Choosing the degree of robustness

The choice of \( \theta \) is crucial since it implicitly defines \( \eta \), determining the set of models around the reference model that the monetary authority considers in the design of the robust policy. Small values of \( \theta \) define a large set of models. Some of these models, very far from the reference model, are highly unlikely descriptions of the economy. Therefore, the robust policy, based on these unlikely models, tends to be implausible. On the other hand, high values of \( \theta \) define an overly small neighborhood around the reference model, leaving out some plausible and interesting alternative specifications.

To guide the choice of \( \theta \), Hansen and Sargent (2008) propose a methodology based on the detection error probability, defined as the probability that a researcher, based on time series of length \( T \) for the outcomes of the model, would infer incorrectly whether the approximating model (model AP) or the worst-case model (model WR) generated these time series. I denote this probability by \( p(\theta) \).

The detection error probability measures the difficulty in distinguishing the approximating model from alternative specifications in the set of models associated with \( \theta \). The idea is that the policymaker desires policies that are robust to
models that are hard to distinguish from the approximating model given a sample of time series observations. These models are the most plausible alternative descriptions of the economy given the available information.

The approximating model is the benchmark in the definition of \( p(\theta) \) because it is the reference model, without misspecification, subject to the robust policy. Since the worst-case model is the base of comparison to compute \( p(\theta) \), inference mistakes related to other models in the \( \eta \)-neighborhood are less costly. Therefore, the definition of \( p(\theta) \) considers the most critical situation in terms of choosing the wrong model.

To calculate the detection error probability, consider \( M \) independent simulations of economic outcomes of length \( T \). These simulated data come from the worst-case model. In this situation, the likelihood associated with models \( AP \) and \( WR \) are \( L_{AP|WR} \) and \( L_{WR|WR} \). The researcher chooses incorrectly model \( AP \) over model \( WR \) if \( \log[L_{WR|WR}/L_{AP|WR}] < 0 \). The probability of choosing \( AP \) given that \( WR \) generates the data is \( p(AP|WR) \) and equals approximately the fraction of the \( M \) simulations that satisfies \( \log[L_{WR|WR}/L_{AP|WR}] < 0 \). Analogously, the probability of choosing \( WR \) given that \( AP \) generates the data, \( p(WR|AP) \), is based on \( M \) draws from the approximating model, using the likelihoods \( L_{WR|AP} \) and \( L_{AP|AP} \).

The average between \( p(AP|WR) \) and \( p(WR|AP) \) is the detection error probability, i.e., \( p(\theta) = (p(AP|WR) + p(WR|AP))/2 \). Following Hansen and Sargent (2008), I choose \( \theta \) by selecting a reasonable number for \( p(\theta) \) and numerically inverting the expression for the detection error probability to find the value of \( \theta \) compatible with the fixed \( p(\theta) \). In short, to compute robust policies, I need to specify the degree of robustness associated with each chosen value for \( \psi \) by fixing three objects: \( p(\theta), M \) and \( T \).

If \( p(\theta) \) is close to zero, model \( AP \) and model \( WR \) would be so distant that they would be easily distinguished based on the available data. Zero robustness corresponds to \( p(\theta) = 0.5 \). Hansen and Sargent (2008) suggest the range between 10% and 20% for \( p(\theta) \). I set the detection error probability at 20%, following the calibrations in the examples described in Hansen and Sargent (2008, Chapters 9 and 10).

My calibration allows deviations from model \( AP \) such that 80% of the time a researcher can distinguish between model \( AP \) and model \( WR \). This is a reasonable figure since the reference model used in the case of no misspecification (model \( AP \) is the monetary authority’s best attempt at describing the economy.

I set \( M = 10,000 \) such that the approximate detection error probability based on \( M \) simulations is very close to the fixed \( p(\theta) \) and does not improve significantly in terms of accuracy in a grid of values for \( M \) from 1000 to 20,000.

Since a large sample \( T \) reduces the uncertainty surrounding the reference model and leads to high values of \( \theta \), I set \( T \) to be on average close to sample sizes found in empirical studies using quarterly macroeconomic data for the U.S. These studies usually cover the period from 1960 to 2010 with 204 observations, or the ‘Great Moderation’ period (from 1984 to 2007) with 96 observations. I set \( T \) as the average between 204 and 96. Thus, the sample size for each simulation run is \( T = 150 \).

My choices for \( p(\theta) \) and \( T \) are in line with Hansen and Sargent (2008, Chapter 14), which describes an asset pricing application. In addition, Giordani and S¨oderlind (2004) specify \( \theta \) such that the detection error probability is 20% for a sample size of 150 periods with \( M = 10,000 \).

Table 1 shows the implied values for \( \theta \) under persistent shocks (P) and white noise shocks (W), according to the choices I made for \( p(\theta), M \) and \( T \).

### 3.3.2. Basic numerical results

I use the baseline calibration to generate numerical results under persistent shocks. However, to facilitate comparison with my previous findings, I also present numerical results under white noise shocks.

The solution, under persistent shocks, can be represented as follows. Since both shocks are state variables, \( u_t = c_1 u_{t-1} + c_2 y_{t-1} + \sigma u_{t-1} \) and \( y_{t} = d_1 u_{t-1} + d_2 y_{t-1} + \sigma_y y_{t-1} \). The values for \( c_1, c_2, d_1 \) and \( d_2 \) depend on the type of equilibrium being considered. The evil agent’s choices are written as \( v_{t} = f_1 u_{t-1} + f_2 y_{t-1} \) and \( v_{t} = g_1 u_{t-1} + g_2 y_{t-1} \). Each forward-looking variable is written as a linear function of the state variables \( u_t \) and \( y_t \). The expressions are \( q_t = a_t u_t + b_t y_t \), \( x_t = a_t^{-1} u_t - b_t y_t \), \( \pi_t = a_t^{-1} u_t + b_t y_t \) and \( r_t = a_t u_t + b_t y_t \).

This representation connecting forward-looking variables to shocks also describes the solution under white noise shocks, but the state variables are \( e_t \) and \( e_t \). Therefore, under white noise shocks, I have \( q_t = a_t e_t + b_t e_t \), \( x_t = a_t e_t + b_t e_t \), \( \pi_t = a_t^{-1} e_t + b_t e_t \) and \( r_t = a_t e_t + b_t e_t \).

The coefficients in these linear rules fully characterize the dynamic behavior of the model. In fact, variances and impulse responses are functions of these coefficients. Next, for a given value of \( \psi \), I present the coefficients in the linear rules connecting the forward-looking variables to the disturbances \( u_t \) and \( y_t \). In addition, I show the laws of motion for \( u_t, y_t, v_t \) and \( v_t \).

<table>
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<th>( \theta )</th>
<th>( \theta )</th>
<th>( \theta )</th>
<th>( \theta )</th>
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Note: P for persistent and W for white noise.
In the approximating model, under persistent shocks, the laws of motion for $u_t, y^n_t, v^n_t$ and $v^n_t$ are characterized by $c = d_1 = 0, f_1 = f_2 = g_1 = g_2 = 0$, and $c_1 = d_2 = 0.9$. In the worst-case model, $c_2 = d_1 = 0, f_1 = 0.054, f_2 = g_1 = g_2 = 0$, $c_1 = 0.954$ and $d_2 = 0.9$. The coefficients $f_1$ and $c_1$ do not depend on $\psi$, being functions of the chosen detection error probability. The linear rules connecting the forward-looking variables to the disturbances $u_t$ and $y^n_t$ are the same in the worst-case model and in the approximating model. However, time series dynamics and dynamic responses to changes in $c^n_t$ differ across the two notions of equilibrium because cost shocks are more persistent in the worst-case model.

The results for robust policies are summarized in Table 2. To clarify how persistence in the shocks changes the results based on white noise shocks, three sets of results are reported together. I assign the label P to persistent shocks, W to the worst-case model and in the approximating model. However, time series dynamics and dynamic responses to changes in $c^n_t$ differ across the two notions of equilibrium (approximating or worst-case model). Under white noise shocks $u_t = \sigma_u (e^n_t + v^n_t)$, $y^n_t = \sigma_n e^n_t$, $v^n_t = 0$ and $v^n_t = (\alpha_d \sigma_u / \theta) a^n_t e^n_t$, where $a^n_t$ is the inflation coefficient associated with the worst-case model in Table 2. In the approximating model, $v^n_t = 0$.

Propositions 1–3 are direct implications of optimality conditions, which are the same irrespective of the degree of persistence in the shocks. Therefore, these propositions are still valid in the case of persistent shocks. In fact, a comparison of the law of motion for $u_t$ in the worst-case model and in the approximating model shows that the evil agent adds noise to cost shocks, as predicted in Proposition 2. In addition, numerical solutions show that the evil agent does not add any noise to shocks to the natural level of output, as stated in Proposition 3. According to Giordani and Söderlind (2004), these are also features of the solution in the standard new Keynesian framework.

Proposition 4 is consistent with numerical results under persistent shocks since inflation increases and the output gap declines after a positive cost shock. However, the absolute values of the coefficients $a_r$ and $a_0$ are bigger under persistent shocks compared to the white noise case. Thus, the presence of persistent shocks magnifies the direct effects of the cost shock on inflation and the output gap.

Propositions 5–8 hinge on the fact that, under white noise shocks, Eq. (1) implies $q_t = -r_t$. This is no longer the case under persistent shocks since the solution method has to take into account the effect of expectations in Eq. (1). In fact, Table 2 shows that equalities $a_0 = -a_r$ and $b_0 = -b_r$ are true only under white noise shocks. Thus, persistent shocks overturn Propositions 5–8.

Under persistent shocks, equity prices still react negatively to cost shocks and interest rates react positively to cost shocks for the range of values considered for $\psi$. Marginal costs increase with positive cost shocks, leading to higher inflation and lower dividends. This fact explains how equity prices and interest rates react to cost shocks. Persistent shocks lead to strong reactions of equity prices to cost shocks compared to the case of white noise shocks. The monetary policy problem is not isomorphic to the analogous problem in the new Keynesian framework. Consequently, equity price dynamics should be explicitly considered in the solution of the robust control problem.

In short, persistence in the shocks overturns Propositions 5–8, numerical solutions conform to Proposition 4 and Propositions 1–3 remain valid statements.

### 3.3.3. Numerical results on the effects of robustness

Under persistent shocks, the parameterization for the non-robust policy case is given by the laws of motion for $u_t, y^n_t, v^n_t$ and $v^n_t$ characterized by $c_2 = d_1 = 0, f_1 = f_2 = g_1 = g_2 = 0$ and $c_1 = d_2 = 0.9$. Under white noise shocks, $u_t = \sigma_u e^n_t$, $y^n_t = \sigma_n e^n_t$.

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Note: P for persistent, W for white noise/worst-case and A for white noise/approximating.
and \( v_t = \alpha = 0 \). The non-robust policy yields the results summarized in Table 3. In parentheses, P and W stand for persistent shocks and white noise shocks.

To gauge if Propositions 9–11 are in accordance with numerical solutions, I compare the coefficients \( a_q \), \( a_x \), and \( a_r \) in Tables 2 and 3.

Proposition 9 is no longer valid under persistent shocks. Though the output gap, interest rate and equity prices respond with more intensity to the cost shock when the preference for robustness increases, the increase in volatility is different across equilibrium concepts since the dynamics of \( u_t \) is more persistent in the worst-case model with \( c_1 > 0.9 \). Therefore, the first part of Proposition 9 concerning the increase in absolute values of \( a_q \), \( a_x \), and \( a_r \) in Table 2 compared to Table 3 is in line with numerical findings, but the second part, which states that the increase in volatility is the same across equilibrium concepts, is not true.

Proposition 10 is consistent with the numerical results since the magnitude of \( a_x \) is bigger in Table 2 compared to Table 3 under persistent shocks. In the worst-case model, regardless of the degree of persistence in the shocks, an increased preference for robustness makes inflation more sensitive to cost shocks.

Proposition 11 is no longer true under persistent shocks. The magnitude of \( a_r \) with label P is bigger in Table 2 compared to Table 3. This pattern contradicts the behavior of \( a_r \) that emerges by comparing values for this coefficient with label A in Table 2 with those with label W in Table 3. In fact, in the approximating model, under white noise shocks, inflation is less sensitive to cost shocks with an increase in the preference for robustness. This is not the case in the approximating model under persistent shocks.

To sum up, persistence in the shocks overturns Propositions 9 and 11 and numerical solutions conform to Proposition 10.

### 3.3.4. Numerical results on the role of wealth effects

Proposition 12 does not hold under persistent shocks because interest rates are decreasing in \( \psi \). This pattern is common to robust and non-robust policies and is related to how persistent shocks affect equilibrium dynamics through expectations.

In fact, if shocks are white noises, their propagation via expectations is absent, thus the monetary authority can always respond more strongly to cost shocks when \( \psi \) increases, affecting equity prices contemporaneously.

However, with persistent shocks, the effect of the shock itself and the monetary policy actions taken today may have repercussions in the future. An adverse cost shock today, due to persistence, may ignite a string of adverse cost shocks which, in turn, will have a negative impact on future marginal costs, finally affecting equity prices through expected dividends. An aggressive response to cost shocks today may signal more interest rate hikes tomorrow and lower future values for the output gap. As a result, expected dividends will be lower, leading to a decline in equity prices.

Persistent cost shocks and aggressive monetary policy may depress the economy, through the effects of equity prices on aggregate demand, beyond the level compatible with the central bank’s preference for output stabilization. Therefore, to avoid a deep economic slowdown, catalyzed by the consumption-wealth effect, the central bank acts softly. The reduction in interest rates as \( \psi \) increases is needed to counteract the negative effects of a drop in equity prices on aggregate demand. According to the numerical results in Tables 2 and 3, under persistent shocks, equity prices are more responsive to cost shocks as a function of \( \psi \) since a less aggressive monetary policy makes expected real marginal costs and dividends more sensitive to cost shocks.

Proposition 13 remains true. For the baseline parameterization, under persistent shocks, \( a_r \) seems to decrease with \( \psi \). Thus, inflation seems to be less responsive to cost shocks as \( \psi \) increases. This is a plausible outcome since inflation, in a new Keynesian Phillips curve specification, is the present discounted value of future real marginal costs. Though expected real marginal costs become more sensitive to cost shocks, the discount factor \( \hat{\beta} \) becomes smaller, making inflation less responsive to cost shocks.

---

**Table 3**

Non-robust policy.

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>( a_q )</th>
<th>( b_q )</th>
<th>( a_x )</th>
<th>( b_x )</th>
<th>( a_r )</th>
<th>( b_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>-0.6270(P)</td>
<td>1(P)</td>
<td>-0.4004(P)</td>
<td>0(P)</td>
<td>0.3066(P)</td>
<td>0(P)</td>
</tr>
<tr>
<td></td>
<td>-0.0971(W)</td>
<td>1(W)</td>
<td>-0.0971(W)</td>
<td>0(W)</td>
<td>0.0744(W)</td>
<td>0(W)</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.6790(P)</td>
<td>1(P)</td>
<td>-0.3988(P)</td>
<td>0(P)</td>
<td>0.3011(P)</td>
<td>0(P)</td>
</tr>
<tr>
<td></td>
<td>-0.0995(W)</td>
<td>1(W)</td>
<td>-0.0995(W)</td>
<td>0(W)</td>
<td>0.0751(W)</td>
<td>0(W)</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.1912(P)</td>
<td>1(P)</td>
<td>-0.3907(P)</td>
<td>0(P)</td>
<td>0.2686(P)</td>
<td>0(P)</td>
</tr>
<tr>
<td></td>
<td>-0.1177(W)</td>
<td>1(W)</td>
<td>-0.1177(W)</td>
<td>0(W)</td>
<td>0.0804(W)</td>
<td>0(W)</td>
</tr>
<tr>
<td>0.1</td>
<td>-1.0370(P)</td>
<td>1(P)</td>
<td>-0.3877(P)</td>
<td>0(P)</td>
<td>0.2386(P)</td>
<td>0(P)</td>
</tr>
<tr>
<td></td>
<td>-0.1393(W)</td>
<td>1(W)</td>
<td>-0.1393(W)</td>
<td>0(W)</td>
<td>0.0857(W)</td>
<td>0(W)</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.1085(P)</td>
<td>1(P)</td>
<td>-0.3929(P)</td>
<td>0(P)</td>
<td>0.1935(P)</td>
<td>0(P)</td>
</tr>
<tr>
<td></td>
<td>-0.1946(W)</td>
<td>1(W)</td>
<td>-0.1946(W)</td>
<td>0(W)</td>
<td>0.0958(W)</td>
<td>0(W)</td>
</tr>
</tbody>
</table>

Note: P for persistent and W for white noise.
However, this pattern is not robust and depends on the chosen parameter values. For instance, fixing $\theta = 50,000, \rho_n = \rho_a = 0.4$ and maintaining the remaining calibrated values, I get $a_c = 0.1127$ for $\psi = 0.01$ and $a_c = 0.1236$ for $\psi = 0.25$, suggesting an increasing pattern for $a_c$. Thus, the impact of a stronger wealth effect on inflation depends on the calibration.

In short, Proposition 12 does not hold while Proposition 13 continues to be a valid statement under persistent shocks.

4. Conclusion

In a simple overlapping generations model with sticky prices, I studied how optimal monetary policy is affected by the central bank’s concerns about model uncertainty. The artificial economy considered allows monetary policy to be transmitted through the consumption-wealth channel. Consequently, equity prices become relevant for the behavior of the economy in response to monetary policy actions.

This essay analyzed how the central bank’s desire to be robust against model misspecification affects the behavior of the economy according to the importance of the size of the wealth effect. To be able to find closed-form solutions, paralleling Leitemo and Söderström (2008a, 2008b), I initially assumed that all shocks were white noises.

Under the assumption of white noise disturbances, the robust policy always responds more aggressively to cost shocks than the non-robust policy. Consequently, inflation is less volatile in the approximating model while the output gap is more volatile. The preference for robustness does not affect the response to shocks to the natural level of output. In addition, equity prices become more sensitive to cost shocks, and as a consequence more volatile under the robust policy.

Concerning the interaction between robustness and the size of wealth effects, monetary policy responses to cost shocks are even more aggressive if the consumption-wealth channel has an important role in the transmission mechanism. The output gap and equity prices become more volatile but the effect on inflation is ambiguous.

I also showed that the monetary policy problem in the overlapping generations model is isomorphic to the same problem in a simple representative agent new Keynesian closed economy with the IS implied by logarithmic preferences, a more volatile Phillips curve disturbance and a larger output gap-inflation tradeoff.

All these findings depend on the assumption of white noise disturbances. Under persistent shocks, I used numerical methods to solve the model. The solution shows that an increase in the preference for robustness also leads to more aggressive responses to cost shocks. In contrast, stronger wealth effects are associated with less aggressive responses. Thus, the presence of the consumption-wealth channel attenuates the effect of model uncertainty on monetary policy stance.

If shocks are short-lived, the central bank can act more aggressively, since the impact of monetary policy on equity prices and the output gap will not depend on additional effects working through expectations. With persistent shocks, the effect of the shock itself and monetary policy actions taken today may have repercussions in the future. An aggressive response to cost shocks today may signal lower values for the output gap and dividends tomorrow, leading to a decline in equity prices, which in turn may depress the economy beyond the level compatible with the preference for output stabilization characterizing the central banker.

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Appendix A

This appendix presents a discrete-time version of the perpetual youth model based upon Blanchard (1985) and Yaari (1965) with sticky prices. The model is a simplified version of Nisticó (2012) with cost shocks and shocks to the natural level of output.

A.1. Population structure and households

At a given time $t$, a new generation of consumers with uncertain lifetimes is born. Let $\gamma$ be the probability of dying before the next period begins.

The size of cohort $s$ at time $t$ is given by $n_{st} = n (1-\gamma)^{t-s}$. In each generation, there is a continuum of consumers indexed by $j$ and uniformly distributed in the interval $[0,1]$. Thus, the aggregate population can be computed as

$$n_t = \sum_{s = -\infty}^{t} n(1-\gamma)^{t-s} \int_{0}^{1} dj = \frac{n}{\gamma}$$

Assuming zero population growth, $n_t$ can be normalized to 1. Therefore, $n = \gamma$ is the size of a new generation born at time $t$. Since population is constant, a fraction of equal size is dying.
I consider a cashless economy in which consumer \(j\) belonging to a generation born at \(s\) faces the following optimization problem:

\[
\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t (1-\gamma)^t [\log(C_t(s,j)) + \kappa \log(1-L_t(s,j))]
\]

subject to the following constraints:

\[
P_t C_t(s,j) + B_t(s,j) + P_t \int_0^1 Q_t(i) Z_t(s,j,i) \, di \leq W_t(s,j) L_t(s,j) + \omega_t(s,j) \tag{8}
\]

\[
L_t(s,j) = \left( \frac{W_t(s,j)}{W_t(s)} \right)^{-\frac{1}{\gamma}} L_t(s) \tag{9}
\]

The choice variables are consumption \(C_t(s,j)\), labor \(L_t(s,j)\), a risk-free bond whose nominal value is \(B_t(s,j)\), and a portfolio of shares \(Z_t(s,j,i)\), whose real price is \(Q_t(i)\), which are issued by a continuum of firms indexed by \(i\).

\(W_t(s,j)\) is nominal wage and \(\omega_t(s,j)\) is the amount of financial wealth belonging to consumer \(j\) from the generation born at \(s\).

\(\beta\) is the subjective discount factor and \(\kappa\) is a preference parameter.

Households have some market power in the labor market. Under monopolistic competition, there are firms able to bundle cohort specific labor in order to maximize the following criterion:

\[
W_t(s,j) L_t(s) - \int_0^1 W_t(s,j) L_t(s,j) \, dj
\]

subject to the technology for bundling labor inputs \(L_t(s,j)\):

\[
L_t(s) = \left[ \int_0^1 L_t(s,j)^{(\gamma_t-1)/\gamma_t} \, dj \right]^{\gamma_t/(\gamma_t-1)}.
\]

The aggregate wage is given by \(W_t = \left[ \int_0^1 W_t(s,j)^{1-\gamma_t} \, dj \right]^{1/(1-\gamma_t)}\).

Since there is no bequest motive and lifetime is uncertain, there is a life insurance market that redistributes among those who survived, in a given cohort, the financial wealth of the members who died. The zero profit condition in the competitive insurance market implies a gross return of \(1/(1-\gamma_t)\). Therefore, financial wealth is given by

\[
\omega_t(s,j) = \frac{1}{1-\gamma_t} \left[ B_{t-1}(s,j) R_{t-1} + P_t \int_0^1 (Q_t(i) + D_t(i)) Z_{t-1}(s,j,i) \, di \right] \tag{10}
\]

\(D_t(i)\) is the dividend paid by the share and \(R_{t-1}\) is a risk-free interest rate.

The first-order conditions are

\[
\frac{W_t(s,j)}{P_t} = \kappa (1+\mu_t^w) \frac{C_t(s,j)}{1-L_t(s,j)} \tag{11}
\]

\[
R_t E_t[A_{t+1}(s,j)] = 1 \tag{12}
\]

\[
Q_t(i) P_t = E_t[A_{t+1}(s,j) P_{t+1}(Q_{t+1}(i) + D_{t+1}(i))] \tag{13}
\]

As in Clarida et al. (2002), \(\mu_t^w = 1/(\gamma_t-1)\) is the exogenously varying wage markup, which is the source of cost shocks. In Eqs. (12) and (13), \(A_{t+1}(s,j)\) denotes the stochastic discount factor which is given by

\[
A_{t+1}(s,j) = \beta \frac{P_t C_t(s,j)}{P_{t+1} C_{t+1}(s,j)} \tag{14}
\]

Human wealth \(H_t(s,j)\), for individual \(j\) from cohort \(s\) is

\[
H_t(s,j) = E_t \sum_{k=0}^{\infty} A_{t+k}(s,j)(1-\gamma)^k W_{t+k}(s,j) L_{t+k}(s,j) \tag{15}
\]

Using Eqs. (8), (10), (13)–(15) and the transversality condition, Piergallini (2006) and Nisticò (2012) show that nominal consumption is a linear function of financial and human wealth, according to

\[
P_t C_t(s,j) = \varphi [\omega_t(s,j) + H_t(s,j)] \tag{16}
\]

where \(\varphi = 1 - \beta (1-\gamma)\) is a constant.
A.2. Aggregation across individuals

Since wages are flexible and the labor market is not segmented by cohort, there is just one single wage. Therefore, \( W_t(s) = W_t \). Since all consumers make the same decision concerning labor supply, wages are equalized across individuals and are given by \( W_t(s,J) = W_t(s) = W_t \).

The aggregate value of any macroeconomic variable, except wages, is given by a weighted average of each consumer \( j \) in each generation \( s \). For the computation of the average, I use the size of each generation and the distribution of agents belonging to generation \( s \).

In fact, if \( x_t(s,j) \) stands for any variable associated with individual \( j \) in a given generation \( s \), the aggregate variable \( X_t \) is given by

\[
X_t = \sum_{j=-\infty}^{1} (1-\gamma)^{j-1} \int_{0}^{1} x_t(s,j) \, dj
\]

Applying the aggregator above to expressions (11)–(13), (8) and (16) yields the following aggregate expressions:

\[
\frac{W_t}{P_t} = \kappa(1+\mu^{\gamma}) \left[ \frac{C_t}{1-L_t} \right] \quad (17)
\]

\[
R_t E_t[A_{t,t+1}] = 1 \quad (18)
\]

\[
Q_t(i)P_t = E_t[A_{t,t+1}P_{t+1}(Q_{t+1}(i) + D_{t+1}(i))] \quad (19)
\]

\[
P_tC_t + B_t + P_t \int_{0}^{1} Q_t(i)Z_t(i) \, di \leq W_tL_t + \omega_t \quad (20)
\]

\[
P_tC_t = \varphi[\omega_t + H_t] \quad (21)
\]

The aggregate wealth is given by

\[
\omega_t = B_{t-1}R_{t-1} + P_t \int_{0}^{1} (Q_t(i) + D_t(i))Z_{t-1}(i) \, di \quad (22)
\]

Piergallini (2006) and Nisticó (2012) show that the Euler equation for aggregate consumption is

\[
P_tC_t = \frac{1}{\beta} E_t[A_{t,t+1}P_{t+1}C_{t+1}] + \frac{\phi^{\gamma}}{(1-\gamma)} E_t[A_{t,t+1}\omega_{t+1}] \quad (23)
\]

A.3. Firms and price-setting behavior

The final good is produced by a representative firm which chooses intermediate goods \( Y_t(i) \) as inputs to maximize profits, subject to the production function \( Y_t = [\int_{0}^{1} Y_t(i)^{v-1} \, di]^{1/(v-1)} \).

The solution of the profit maximization problem yields the following expression for the selling price of the final good:

\[
P_t = \left[ \int_{0}^{1} P_t(i)^{1-v} \, di \right]^{1/(1-v)} \quad (24)
\]

The economy has a continuum of monopolistic firms, uniformly distributed in the interval \([0,1]\). Each firm produces a differentiated intermediate product \( i \), using the linear production function:

\[
Y_t(i) = A_tL_t(i) \quad (25)
\]

where \( A_t \) is an aggregate technology shock and \( L_t(i) \) stands for labor input.

Each firm participates in a competitive labor market and demands labor input such that total real costs \( (W_t/P_t)L_t(i) \) are minimized subject to (18). Real marginal costs are given by \( MC_t = W_t/P_tA_t \).

As in Calvo (1983), in each period firms adjust their prices with a constant and exogenous probability \((1-\phi)\). With probability \( \phi \), a particular firm does not receive the green light to adjust its price and charges the last period price.

The optimization problem is

\[
\text{Max}_{P_t(i)} \left\{ \sum_{k=0}^{\infty} A_{t+k}C \right\}^{1/\phi} \left\{ \frac{P_t(i)}{P_{t+k}} Y_{t+k}(i) - \frac{W_{t+k}}{P_{t+k}} L_{t+k}(i) \right\} \]

The first-order condition for the solution implies the following expression for the optimal price:

\[ P_n = \left( \frac{V}{V-1} \right) E_t \left\{ \sum_{k=0}^{\infty} \frac{\phi^k A_{t+k}}{E_t} Y_{t+k} [P_{t+k-1}^{n-1} - P_{t+k}^{n-1}] P_{t+k-1}^{n-1} M_{t+k} \right\} \]  

Using Eq. (17), the aggregate price level can be written as

\[ P_t = (1-\phi)P_t^{n-1} + \phi P_{t-1} \]

A.4. Equilibrium

The market-clearing conditions for bonds, equities and final goods are

\[ B_t = \sum_{s=-\infty}^{1} \gamma(1-\gamma)^{s-1} \int_0^1 B_t(s,j) \, dj = 0 \]

\[ Z_t(i) = \sum_{s=-\infty}^{1} \gamma(1-\gamma)^{s-1} \int_0^1 Z_t(s,j,i) \, dj = 1 \]

\[ Y_t = C_t \]

Aggregate dividends and the equity price are

\[ D_t = \int_0^1 D_t(i) \, di \]

\[ Q_t = \int_0^1 Q_t(i) \, di \]

After imposing market-clearing conditions and \( L_t = Y_t / A_t \), expressions (17), (19), (22) and (20) become

\[ \frac{W_t}{P_t} = k(1+\mu_t^w) \left[ \frac{Y_t}{1 - \left( \frac{Y_t}{A_t} \right)} \right] \]

\[ Q_t P_t = E_t[A_{t+1}P_{t+1}(Q_{t+1} + D_{t+1})] \]

\[ \omega_t = P_t Q_t + P_t D_t \]

\[ P_t Y_t = W_t \frac{Y_t}{A_t} + P_t D_t \]

Using (22) and (23), \( E_t[A_{t+1} \omega_{t+1}] = P_t Q_t \). Accordingly, a new version for the aggregate Euler equation (16) is

\[ P_t Y_t = \frac{1}{\beta} E_t[A_{t+1}P_{t+1}Y_{t+1}] + \frac{\phi^\gamma}{(1-\gamma)} Q_t \]

Since the Euler equation for aggregate consumption depends on aggregate financial wealth, monetary policy can be transmitted through the consumption-financial wealth channel.

Finally, Eqs. (18), (26)–(29), (31) and (32) characterize the equilibrium.

Log-linearizing the equilibrium equations around the steady state yields

\[ q_t = \beta E_t(q_{t+1}) + (1-\beta)E_t(d_{t+1}) - [r_t - E_t(\pi_{t+1})] \]

\[ y_t = \frac{\psi}{1+\psi} q_t + \frac{1}{1+\psi} E_t(q_{t+1}) - \frac{1}{1+\psi} [r_t - E_t(\pi_{t+1})] \]

Lower-case letters denote the difference between a given economic variable and its steady state, both measured in logarithms. The variable \( u_t \) is the log-linear approximation of \( 1+\mu_t^w \) and \( \pi_t \) represents inflation. In addition, \( \mu = \sqrt{V-1} \) and \( \chi = L/(1-L) \), where \( L \) denotes the steady state labor.

The log-linear approximations for real marginal costs and dividends are \( mc_t = (1+\chi)(y_t-a_t) + u_t \) and \( d_t = y_t + 1/(1-\mu)mc_t \).
The log-linear approximation of the natural level of output is $y_t = a_t$. The output gap is $x_t = y_t - y^*_t$. By using these definitions and the log-linear expressions for real marginal costs and dividends, Eqs. (33) and (34) become Eqs. (1) and (2) in Section 2.

References


